nature portfolio

Peer Review File

Classically squeezed multi-mode phonon lasers with levitated optomechanics

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This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
The manuscript entitled “Squeezed multi-mode phonon lasers with levitated optomechanics” reports a phonon laser formed by an optically levitated microparticle dissipatively coupled to an active optical cavity and focuses specifically on its noise squeezing in the multimode regime.
The experimental platform is undoubtedly impressive; the authors have previously demonstrated this setup in several high-impact publications, marking a significant milestone in levitated optomechanics. However, while the technical execution is sound, the conceptual advancement in this current submission appears limited. The primary methodology for squeezing—utilizing stroboscopic switching of the potential—is a well-established technique that has been explored in the literature for over a decade and required an addition of a controlled AOM to the previous setup.
In its current form, the manuscript combines two existing methodologies. While this is a logical progression, I find it an incremental step rather than an important advance of significance to specialists expected for Nature Communications.
Although I cannot recommend the paper for publication in its current state, the following points should be addressed to improve the clarity and rigor of the work for future submissions:
• On p.3, l.64, the authors describe the transition from thermal motion to coherent oscillations as the hallmark of a phonon laser. It would strengthen the discussion to reference and differentiate this from similar transitions reported in several experimental non-Hermitian systems, where levitated microparticle(s) form limit cycles.
• The active cavity is visually present in Figure 1, but its specific role and functionality are not addressed in the text or the legend.
• Variances \sigma^2_i and \sigma^2_0 are critical parameters in this study, however they are not clearly defined in the manuscript – in the caption of Fig.1 it is stated “\sigma^2_i is calculated from the cross-section of the phase-space distributions on q=0” (but there are two maxima for positive and negative p), in SI p.8 line 138 as “the momentum variance of the initial state and the squeezed state” (that are of ring-like form and non-Gaussian).
• Please include the specific wavelength of the trapping laser, as this is a fundamental parameter for reproducibility.

Reviewer #2

(Remarks to the Author)
The manuscript “Squeezed multi-mode phonon lases with levitated optomechanics” by Song et al. reports the experimental realisation of quadrature squeezing in a levitated optomechanical phonon-lasing system. The authors use non-adiabatic trap-frequency changes induced by an AOM-modulated trapping beam to squeeze a self-oscillating microsphere and claim 3.15 dB squeezing for the fundamental phonon-laser mode and 1.23 dB for the second-harmonic mode, at room temperature and low vacuum (~20 mbar). The central claimed advance is the first demonstration of a squeezed nonlinear phonon laser, together with concurrent control of multiple phonon-laser modes.

The work has several clear strengths. First, the experiment appears technically nontrivial: the authors work with a levitated microsphere in an active cavity and exploit a transition into sustained coherent oscillation, i.e. phonon lasing. Second, the squeezing protocol is conceptually simple and experimentally elegant, relying on abrupt frequency shifts rather than complicated feedback. Third, the extension from the fundamental mode to the second harmonic is interesting and could indeed be useful in future multimode sensing or nonlinear phononics applications.

While the experiment is interesting, I do not find that the manuscript, in its present form, meets the standard of Nature Communications. My main concern is that the central claims are stronger than what is actually demonstrated. The unnecessary overselling argumentation is also evident. The work shows quadrature-noise reduction in a strongly thermal, highly occupied regime (the manuscript itself refers to phonon number $\sim 10^{10}$), but the discussion repeatedly pushes toward the quantum-classical boundary, macroscopic superposition, and even gravitational-wave detection (in the conclusions). That framing is not supported by the data presented here.

Major concerns:

1. I believe that the manuscript does not clearly distinguish classical thermomechanical squeezing from genuinely quantum squeezing. The language of the paper is frequently quantum-optical, with operator formalism, Bogoliubov transformations, and claims about the "quantum-classical boundary." At the same time, the experiment is performed with a microsphere at room temperature, low vacuum, and very large phonon number ($\sim 10^{10}$). In this regime, what is demonstrated is best described as quadrature-noise reduction of a classical or thermomechanical self-oscillator, not nonclassical squeezing below the zero-point level. The manuscript should state this explicitly and avoid wording that could be read as a claim of quantum squeezing. I would say the manuscript is very ambiguous on this point. In particular, the discussion linking the present measurements to the "fundamental quantum-classical boundary" is too strong in the absence of a quantum calibration.

2. For the fundamental mode, the authors use g_2 to argue that coherence is preserved over the pulse-width scan, and they report values near 1 for moderate conditions. However, they also show that coherence degrades sharply when the power ratio becomes too large. For the second harmonic, g_2 remains around 1.2, which already indicates inferior coherence compared with the fundamental mode. The manuscript should define more carefully what criterion is being used to call the second harmonic a "phonon laser" during the squeezing protocol. An actual clear definition of phonon laser is missing in the text (see comment below). It would help to include linewidths, phase diffusion, power spectra before and after the pulse, or a more systematic characterization of coherence across the operating regime used for the headline squeezing values. Overall, the evidence that the system remains a coherent phonon laser throughout the squeezing protocol is not fully convincing.

3. The key observable is the ratio σ_t^2/σ_0^2, extracted from repeated pulse sequences and phase-space reconstruction. The Methods state that 2500–3000 pulses were used per parameter set, and that each setting was repeated 20 times with standard-deviation error bars. This is useful, but still insufficiently documented in the main text. The authors should explain in more detail how the specific phase is selected for the variance extraction, how the phase drift is handled from shot to shot, and whether the reported minimum corresponds to a single time bin, an average over a phase window, or a fit. It would also be important to quote uncertainties directly on the dB squeezing values, not only on the raw variance points. As it stands, the difference between a result marginally above 3 dB and one at the 3 dB level is not rigorously established.

4. The phase-space distributions rely on estimating momentum by differentiating measured position traces. The authors note that the time resolution is 1 μs, much shorter than the oscillation period and momentum relaxation time, and therefore infer that momentum can be estimated reliably. That is a reasonable starting point, but numerical differentiation is notoriously noise-sensitive. The manuscript should provide a control analysis showing that the apparent squeezing is not distorted by derivative filtering, finite bandwidth, or detector noise. A calibration using synthetic signals or an independently validated estimator for the quadratures would make the result much more robust.

5. The theoretical description is too compressed and, in places, unclear. The non-adiabatic frequency-jump protocol is familiar, but here it is being applied to a self-oscillating nonlinear phonon-laser state. The manuscript jumps rather quickly from the switching of trap power to a harmonic-oscillator squeezing operator and a simple formula for the dB reduction. The authors should better justify the effective Hamiltonian used during the jump, state the assumptions under which the standard squeezing transformation applies, and explain how the harmonic component inherits squeezing from the same protocol. Eq. (5), which relates g_2 to quadrature variables, is also difficult to parse in its present form and should be rewritten more carefully.

6. There are already important prior demonstrations of squeezing in levitated optomechanics, including thermal and quantum squeezing via fast frequency changes, and the authors themselves previously reported nonlinear multi-frequency phonon lasers. The true novelty here appears to be the application of such squeezing to a self-oscillating phonon-lasing state, plus the observation of a squeezed second harmonic. That is interesting, but the manuscript should articulate this more precisely and compare directly with the closest prior works: the novelty relative to prior work needs to be sharpened. Right now, the introduction and discussion sometimes blur together squeezing of levitated oscillators in general and squeezing of a phonon laser in particular. A more focused positioning would strengthen the paper.

Style comment:

-) The manuscript lacks a broad audience introduction about phonon laser. The introduction is a mere list of previous works and possible applications. This is not conformal to the style of Nature Communications.

-) Several claims about applications are overstated. The concluding discussion mentions macroscopic superposition, gravitational-wave detection, and broadly enhanced precision metrology. These (over)claims are not really supported by the present data. The experiment is performed at room temperature and 20 mbar, with squeezing persisting for only a few ms for the fundamental mode and less than 1 ms for the second harmonic. This is a respectable proof of principle, but it is still far from the mentioned applications. The discussion should be toned down and kept closer to what is actually demonstrated and realistic in the near future.

Minor comments.

-) The manuscript requires editing. There are many awkward or ungrammatical phrases, for example “more exacerbated,” “uses in a huge phonon number,” “coherence fails to retain,” and several figure-caption sentences that are syntactically broken. The adjective “huge” is used several time and it sounds non-scientific (huge with respect to which physical quantity?)

-) The statement that the protocol is “inherently free from the 3 dB squeezing limit” is too quick and needs a clearer theoretical basis, especially since the reported value only marginally exceeds 3 dB.

-) The manuscript should clarify whether the “simultaneous” squeezing of the fundamental and second-harmonic modes means truly simultaneous joint observation in the same run, or simply that the same protocol can be applied to both modes under the same platform. The present figures show separate analyses rather than an explicitly joint measurement.

-) Maybe it would help the reader to include one panel with representative raw time traces before and after the squeezing pulse, not only reconstructed phase-space clouds.

In summary, this is a promising experimental study, but the current manuscript overstates its implications and does not yet provide a sufficiently rigorous quantitative analysis of the squeezing measurement and its interpretation. I would therefore recommend major revision. With a clearer distinction between classical and quantum squeezing, stronger validation of the variance extraction and coherence claims, improved theory exposition, and a more restrained discussion of applications, the manuscript could become suitable for publication.

Reviewer #3

(Remarks to the Author)
The manuscript by Song et al. reports the realization of squeezed phonon lasers. It shows modest (up to ~3 dB) squeezing of the thermal noise of a phonon laser realized in an active optomechanical cavity. While the experimental setup and its ability to realize phonon lasing was described in earlier work (reference 29), the novelty of the current manuscript lies in the fact that the thermal noise of this multimode phonon lasing platform with a large mass is now being squeezed, and in fact in two mechanical modes simultaneously.

I find the work generally to be of interest, and recognize a degree of novelty for the above reasons. Unfortunately, the manuscript does not explain however what the value is of squeezed phonon lasers (either over squeezed thermal or vacuum states or over coherent-state phonon lasers), and thus how this constitutes a significant advance that warrants dissemination to a broad readership. The introduction and conclusions are vague in describing the benefits of squeezed lasing, going no further than "squeezing in phonon lasers is significant because it could reduce thermal noise and increase coherence" and "the thermal noise of the nonlinear phonon lasers is suppressed effectively". Such descriptions are too simple and factually incorrect: Squeezing reduces the fluctuations in one degree of freedom at the expense of increasing the fluctuations in another degree of freedom. As such, one cannot say that it "reduces thermal noise", and it is also not clear what specific applications benefit from thermally squeezed phonon lasing, in comparison to e.g. coherent phonon lasing with reduced fluctuations in both quadratures or the original lasing state. This complete lack of description of significance and prospect make the manuscript in its current form unsuitable for Nature Communications.
Moreover, the manuscript is unclear in distinguishing between quantum squeezing and (classical) thermal squeezing. The reported experiments concern the latter and are far from the quantum squeezing regime. Still, in the introduction this distinction is completely ignored when comparing to other work on squeezed mechanical states (including true quantum squeezing). This is inappropriate, and does not do justice to the efforts of others to achieve significant quantum squeezing, which is arguably a much more challenging task. (The authors suggest that squeezing is more impressive with a lot of thermal noise, but that is incorrect.) This manuscript should explicitly refer to the observations as thermal squeezing, in the title, the abstract, the introduction, and the conclusions, since the single word 'squeezing' is too ambiguous without proper context and explanation. Moreover, the statement "It provides a powerful capability for probing the fundamental quantum-classical boundary." should be removed, since it does not make sense in light of the above. In fact, I see no impact of this work on quantum science in its current form.

In addition, I have several smaller questions and observations:
- In Figure 1a and Appendix Figure S1, the authors present the measured PSD, which appears to contain residual modes beyond the fundamental mode and the second harmonic. Could the authors comment on the origin and influence of these additional modes? They seem more pronounced at trapping power P_1, while they appear to be less prominent at P_2. How do these modes affect the reported results?
- In Figure 1d, the authors present the state before the squeezing pulse. From the small oscillations visible in the plot, it appears that the state may already be slightly squeezed prior to the application of the squeezing pulse. Could the authors comment on how their results would change if the initial state were a perfectly thermal state?
- In Table S1, the authors note that the mass of the oscillator is approximately 10^{-14} kg. Could the authors comment on how the uncertainty in the oscillator mass propagates to the uncertainties in the p and q axes in Figures 1, 2, and 3?
- In line 53, the authors report a maximum squeezing of 3.15 dB for the fundamental mode and 1.23 dB for the second

harmonic. From Figures 2b and 3c, it appears that these degrees of squeezing are only achieved when using different pulse lengths for the two modes. Could the authors comment on the practicality of using two different pulse lengths if this protocol were applied in sensing or other applications? Additionally, could the authors comment on the maximum degree of squeezing that is achieved when using a single pulse length for both modes?

Version 1:

Reviewer comments:

Reviewer #2

(Remarks to the Author)
I thank the authors for the detailed and careful reply. In my view, the authors have addressed the core points I raised and have substantially improved the paper. The main experimental claims are now sufficiently clear and supported by the data. The work reports the classical squeezing of a nonlinear phonon laser in a levitated optomechanical system, including squeezing of both the fundamental mode and the second-harmonic mode. The experiment is technically interesting, and the combination of phonon lasing, levitated optomechanics, non-adiabatic frequency modulation, and multimode nonlinear mechanical dynamics makes the result suitable for publication in Nature Communications.

To recommend the publication to Nature Communications I need the authors to include the wording classical or thermomechanical squeezing explicitly in the title.
Besides that, I do not think that another round of external review is necessary. Nevertheless I have minor concerns to be addressed:

1. Clarify the novelty claim.
The manuscript still uses broad statements such as “the first experimental demonstration” of a squeezed phonon laser. Given the note added in proof and the existence of a closely related recent experiment on a thermomechanically squeezed phonon laser, the novelty claim should be made more precise throughout the abstract, introduction, and discussion. A more accurate formulation would emphasize the specific contribution of the present work: single-mode classical squeezing of a nonlinear phonon laser in a levitated microsphere, including squeezing of the second-harmonic mode and operation under low-vacuum conditions.

2. Clarify the meaning of “simultaneous” multimode squeezing.
The authors state that the fundamental and second-harmonic modes are squeezed simultaneously. However, the optimal pulse widths for the two modes are different. The text should explicitly distinguish between “both modes are affected by the same modulation protocol” and “both modes reach their optimal squeezing at the same pulse width,” which does not appear to be the case. If the authors wish to retain the word “simultaneous,” they should provide a precise quantitative statement of the squeezing observed for both modes under the same pulse conditions.

3. Check the equations and notation in the squeezing protocol.
Equation (1), the definition of the squeezing operator, and Eq. (2) should be checked carefully for typesetting, signs, and numerical factors. The relation between the frequency ratio, the squeezing parameter, and the squeezing expressed in dB should be written in a transparent way, especially because the experimental value is compared directly with the theoretical expectation. The notation involving Ω_1, Ω_2, $2\Omega_1$, T_1, and T_2 should also be made unambiguous in the discussion of the second harmonic.

4. Avoid using ($g^{(2)}(0)$) as evidence of squeezing.
The variance reduction is the evidence for squeezing. The role of ($g^{(2)}(0)$) is instead to show that the phonon-laser coherence is preserved, or at least monitored, during the modulation protocol. The corresponding wording should be revised.

5. Moderate the statement about the 3 dB limit.
The claim that the method is free from the 3 dB limit should be phrased carefully. The measured fundamental-mode squeezing is only slightly above 3 dB within the stated uncertainty, while the second-harmonic squeezing is below 3 dB. The authors may state that the protocol is not fundamentally constrained by the usual steady-state 3 dB parametric-squeezing limit, but they should avoid implying that the experiment has conclusively demonstrated strong squeezing far beyond this limit.

6. Language and typographical corrections.
The manuscript still contains several grammatical and typographical issues. Examples include awkward phrases such as “a squeezing and anti-squeezing processes,” “a high degree of squeezing occurs,” “different pulse width,” and the typo “unqueened phonon lasers.” These should be corrected before publication.

Overall, I find the revised manuscript scientifically sound and timely. The remaining issues concern precision of presentation rather than the validity of the main result. I recommend acceptance after the above minor revisions and, above all, revision of the the title (I do not need to see this manuscript again).

Reviewer #3

(Remarks to the Author)
The authors have substantially revised their manuscript, removing the problematic statements on impact and signficance, and clarified the fact that their work is in the classical domain. They have also addressed my technical questions. I remain not fully convinced that the demonstrated squeezing can have an actual benefit to sensing applications, since I believe it may effectively reduce interesting signals by similar degree as the thermal noise that is squeezed. That said, I understand that that is an open debate and the authors are not the only ones making such arguments. This paper does contribute to the body of work on squeezing in classical resonators, in this case a phonon laser. It could thus indeed be of sufficient significance for publication in Nature Communications. My earlier objections were taken away by the removal of the over-selling arguments.

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On behalf of all the authors, we would like to express our sincere gratitude to both reviewers for their valuable and constructive comments. Addressing their concerns has significantly enhanced both the quality and readability of our manuscript. Below, we provide a detailed point-by-point response to each comment.

Responses to the comments of Reviewer #1

We thank Reviewer #1 for carefully reading our manuscript and for the helpful comments and constructive suggestions, according to which we have carefully revised our manuscript by addressing all of them in the revised manuscript (as marked in red font).

Comment #0. The manuscript entitled "Squeezed multi-mode phonon lasers with levitated optomechanics" reports a phonon laser formed by an optically levitated microparticle dissipatively coupled to an active optical cavity and focuses specifically on its noise squeezing in the multimode regime.

The experimental platform is undoubtedly impressive; the authors have previously demonstrated this setup in several high-impact publications, marking a significant milestone in levitated optomechanics. However, while the technical execution is sound, the conceptual advancement in this current submission appears limited. The primary methodology for squeezing—utilizing stroboscopic switching of the potential—is a well-established technique that has been explored in the literature for over a decade and required an addition of a controlled AOM to the previous setup.

In its current form, the manuscript combines two existing methodologies. While this is a logical progression, I find it an incremental step rather than an important advance of significance to specialists expected for Nature Communications.

Reply #0. Thank you for recognizing the technical quality of our platform. We appreciate your comment that the squeezing method (stroboscopic potential switching) has been explored. However, we respectfully argue that the novelty of our work lies not in the technique itself, but in its first application to a nonlinear phonon-lasing system with simultaneous squeezing of two mechanical modes.

As Reviewer #2 noted, *"The true novelty here appears to be the application of such squeezing to a self-oscillating phonon-lasing state, plus the observation of a squeezed second harmonic."* Reviewer #3 also stated that *"the novelty lies in the fact that the thermal noise of this multimode phonon lasing platform ... is now being squeezed, and in fact in two mechanical modes simultaneously."* We fully agree with these assessments.

The key to achieving such squeezing lies in the coexistence of nonlinear optical gain and pulse modulation. Accordingly, we have succeeded, for the first time, in demonstrating a classically squeezed nonlinear phonon laser.

We note that the stroboscopic potential switching technique has been successfully applied to generate thermal squeezed state in levitated optomechanics with a nanosphere [Rashid et al., *Phys. Rev. Lett.* **117**, 273601 (2016)]. Our work applies the same switching technique to a microsphere, which operates in a nonlinear phonon laser with higher phonon occupancy.

Very recently, a two-mode thermomechanically squeezed phonon laser was reported in a passive optomechancial system with a trapped nanosphere [K. Zhang etal., *Nat. Commun.* **17**, 2345-(2026)]. Our work presents a complementary proposal based on an active optomechancial system where a microscale sphere is effectively squeezed. Especially, we demonstrate simultaneous squeezing of both the-fundamental and the second-harmonic phonon lasers for the first time.

Our method is highly efficient, requiring neither complex experimental setups nor feedback control technology. It provides the first demonstration of classical squeezing in a nonlinear phonon laser, simultaneously in the fundamental mode and its harmonic. This offers new resources for multimode sensing, nonequilibrium dynamics, and mesoscopic classical thermodynamics.

We have revised the whole manuscript to better articulate these points. We hope this clarifies the significance of our contribution. Thank you again for your valuable feedback.

Although I cannot recommend the paper for publication in its current state, the following points should be addressed to improve the clarity and rigor of the work for future submissions:

Comment #1. On p.3, l.64, the authors describe the transition from thermal motion to coherent oscillations as the hallmark of a phonon laser. It would strengthen the discussion to reference and differentiate this from similar transitions reported in several experimental non-Hermitian systems, where levitated microparticle(s) form limit cycles.

Reply #1. Thank you for your insightful suggestion. We agree that it is valuable to discuss the phase transition observed in our system within the broader framework of limit cycles in non-Hermitian systems. In the revised manuscript (p.3, l.68-71), we have added a discussion comparing our phonon lasers with recent experimental realizations of limit cycles in levitated optomechanical systems [Reisenbauer et al., *Nat. Phys*. 20, 1629 (2024); Liška et al., *Nat. Phys*. 20, 1636 (2024)].

Specifically, we now note that those non-Hermitian platforms demonstrate limit-cycle oscillations via non-reciprocal optical interactions. Our system instead features a self-sustained coherent oscillation arising from dissipative coupling to an active cavity, which establishes an intrinsic gain–loss balance. Moreover, the optical-gain-enhanced nonlinearity in our system enables the generation of high-order mechanical harmonics, while this feature has not been presented in previous non-Hermitian systems. This comparison helps to highlight the unique characteristics of our nonlinear phonon lasers and its relevance for multimode squeezing. We believe this will strengthen the discussion as suggested.

Comment #2. The active cavity is visually present in Figure 1, but its specific role and functionality are not addressed in the text or the legend.

Reply #2. We thank the reviewer for pointing this out. We agree that the role and functionality of the active cavity shown in Figure 1 were not sufficiently addressed in the original manuscript (p.3, l.62). To clarify this, we have added a detailed description in the main text (p.3, l.64-67, p.11, l.268-275).

We now provide a detailed description of the active cavity and its role in our system. The active cavity serves as the critical element that enables dissipative coupling to the trapped microsphere. Unlike the dispersive coupling, dissipative coupling provides a direct channel for energy exchange between the cavity mode and the mechanical oscillator, allowing efficient amplification of mechanical motion. However, strong cavity dissipation leads to a short photon lifetime, whereas efficient optomechanical interaction requires a sufficiently long photon lifetime. These two requirements are fundamentally conflicting in typical passive cavity platforms [J.-W. Meng et al., *Phys. Rev. Lett.* **129**, 073901 (2022)]. In our previous work [Ref. 13], we overcame this limitation by incorporating a Yb^{3+}-doped gain fiber into the cavity. This provides high cavity finesse and tunable gain, maintaining the stronger optomechanical coupling for coherent vibrational amplification. This unique capability distinguishes our platform from other optomechanical systems.

Comment #3. Variances \sigma^2_i and \sigma^2_0 are critical parameters in this study, however they are not clearly defined in the manuscript – in the caption of Fig.1 it is stated "\sigma^2_i is calculated from the cross-section of the phase-space distributions on q=0" (but there are two maxima for positive and negative p), in SI p.8 line 138 as "the momentum variance of the initial state and the squeezed state" (that are of ring-like form and non-Gaussian).

Reply #3. We thank the reviewer for pointing out the unclear definition of the variances σ_t^2 and σ_0^2. In the revised manuscript, we use the measured ratio σ_t^2/σ_0^2 to evaluate the degree of squeezing, where σ_0^2 is the momentum noise variance before the pulse and σ_t^2 denotes the variance at time *t*. We corrected the term "the momentum variance" to "the momentum noise variance" to better reflect the measured quantity.

The noise variances are obtained from the noise distributions of the limit cycle oscillations, which follow Gaussian statistics. The detailed calculation method is described in the Supplementary Information (Section 4. p. 10-11), and the corresponding noise distributions are shown in Fig. S6. We have also clarified this point in the caption of Fig. 1 to ensure consistent definitions.

Comment #4. Please include the specific wavelength of the trapping laser, as this is a fundamental parameter for reproducibility.

Reply #4. We thank the reviewer for this helpful suggestion. We agree that specifying the trapping laser wavelength is essential for reproducibility. Accordingly, we have revised the experimental description in the manuscript, including the trapping laser wavelength (532 nm), the pump laser wavelength (976 nm), and the optical power (1.5mW) (page 12 in the Maintext).

Responses to the comments of Reviewer #2

We are grateful to Reviewer #2 for the careful reading and the insightful comments. In response, we have thoroughly revised the manuscript and highlighted all changes in red.

Comment #0. The manuscript "Squeezed multi-mode phonon lases with levitated optomechanics" by Song et al. reports the experimental realisation of quadrature squeezing in a levitated optomechanical phonon-lasing system. The authors use non-adiabatic trap-frequency changes induced by an AOM-modulated trapping beam to squeeze a self-oscillating microsphere and claim 3.15 dB squeezing for the fundamental phonon-laser mode and 1.23 dB for the second-harmonic mode, at room temperature and low vacuum (~20 mbar). The central claimed advance is the first demonstration of a squeezed nonlinear phonon laser, together with concurrent control of multiple phonon-laser modes.

Reply #0. We are sincerely grateful to the reviewer for taking the time to carefully review our work and point out the key elements of our study.

Our proposal is highly efficient, requiring neither complex experimental setups nor feedback control technology. It provides the first demonstration of classical squeezing in a nonlinear phonon laser, simultaneously in the fundamental mode and its harmonic. This offers new resources for multimode sensing, nonequilibrium dynamics, and mesoscopic classical thermodynamics.

Comment #1. The work has several clear strengths. First, the experiment appears technically nontrivial: the authors work with a levitated microsphere in an active cavity and exploit a transition into sustained coherent oscillation, i.e. phonon lasing. Second, the squeezing protocol is conceptually simple and experimentally elegant, relying on abrupt frequency shifts rather than complicated feedback. Third, the extension from the fundamental mode to the second harmonic is interesting and could indeed be useful in future multimode sensing or nonlinear phononics applications.

Reply #1. We thank the reviewer for recognizing the three clear strengths of our work. As the reviewer pointed out, our work has several notable features. First, we appreciate the acknowledgment that the experiment is technically nontrivial, involving a levitated microsphere in an active cavity and the exploitation of the phonon-lasing transition. Second, we are glad that the reviewer finds our squeezing protocol conceptually simple and experimentally elegant, relying on abrupt frequency shifts rather than complicated feedback. Third, we thank the reviewer for noting that the extension from the fundamental mode to the second harmonic is interesting and potentially useful for future multimode sensing or nonlinear phononics applications.

Comment #2. While the experiment is interesting, I do not find that the manuscript, in its present form, meets the standard of Nature Communications. My main concern is that the central claims are stronger than what is actually demonstrated. The unnecessary overselling argumentation is also evident. The work shows quadrature-noise reduction in a strongly thermal, highly occupied regime (the manuscript itself refers to phonon number ~10^10), but the discussion repeatedly pushes toward the quantum-classical boundary, macroscopic superposition, and even gravitational-wave detection (in the conclusions). That framing is not supported by the data presented here.

Reply #2. We sincerely thank the reviewer for their careful reading and constructive feedback. We agree that the original manuscript overreached by invoking quantum applications such as macroscopic superposition and gravitational wave detection. These claims are not supported by our current experiment, which involves the huge phonon numbers. We have therefore removed them entirely from the revised manuscript. The revised manuscript now focuses more clearly on the demonstrated strengths of our work.

In the revised manuscript (line 224-234 in the Maintext), we have restructured the discussion around the precision enhancement in classical phonon laser sensors, studies of nonequilibrium dynamics, and the classical thermodynamics. The revised conclusion now focuses on what we actually demonstrated. With the non-adiabatic frequency jumping, the momentum noise of the microsphere can be squeezed at the expense of position noise. It provides the valuable resource for sensing applications, including mass sensing, zeptonewton-scale force detection, and so on.

We also give a discussion for squeezed phonon laser combs, which can be applied to boost the research in nonlinear phononics and multimode sensing.

Major concerns:

Comment #3. 1. I believe that the manuscript does not clearly distinguish classical thermomechanical squeezing from genuinely quantum squeezing. The language of the paper is frequently quantum-optical, with operator formalism, Bogoliubov transformations, and claims about the “quantum-classical boundary.” At the same time, the experiment is performed with a microsphere at room temperature, low vacuum, and very large phonon number (~10^10). In this regime, what is demonstrated is best described as quadrature-noise reduction of a classical or thermomechanical self-oscillator, not nonclassical squeezing below the zero-point level. The manuscript should state this explicitly and avoid wording that could be read as a claim of quantum squeezing. I would say the manuscript is very ambiguous on this point. In particular, the discussion linking the present measurements to the "fundamental quantum-classical boundary" is too strong in the absence of a quantum calibration.

Reply #3. We thank the reviewer for raising this problem. We agree that the original manuscript did not clearly distinguish classical thermomechanical squeezing from quantum squeezing, as well as the overreaching description about the "fundamental quantum-classical boundary. We have now addressed these issues point by point.

1. We agree that our experiment operates in a regime of large phonon number ($\sim 10^{10}$) and room temperature. The observed squeezing is classical in nature, originating from the high occupation of the phonon lasers. The description about the quantum squeezing is modified within the whole manuscript.

2. We have removed the phrase "fundamental quantum-classical boundary" from the manuscript (previously on page 11, line 193). All other wording that involves the conception about the quantum squeezing has been eliminated.

3. The key advance is now stated clearly: demonstrating quadrature-noise reduction (squeezing) in a multimode phonon laser. This capability is directly relevant for enhancing the sensitivity of classical phonon laser sensors, such as force or mass sensing (e.g., Zheng et al., *arXiv*:**2604**.06923 (2026); Pan et al., *Laser Photonics Rev.* **18**, 2400593 (2024)). By suppressing momentum noise at the expense of position noise, our method offers a practical route to improve measurement precision at room temperature.

4. We acknowledge that there are no quantum effects in our results due to the large thermal fluctuations. This does not conflict with the theory in quantum-optics framework, which describes the motion of mechanical oscillator from the view of the energy and illustrates the squeezing mechanism of the frequency jumping. This theoretical description is also consistent with prior work on thermomechanical squeezing (e.g., Rashid et al., Phys. Rev. Lett. 117, 273601 (2016)). For clarity, we have refined the theory in Supplementary Section 2.

Comment #4. 2. For the fundamental mode, the authors use g_2 to argue that coherence is preserved over the pulse-width scan, and they report values near 1 for moderate conditions. However, they also show that coherence degrades sharply when the power ratio becomes too large. For the second harmonic, g_2 remains around 1.2, which already indicates inferior coherence compared with the fundamental mode. The manuscript should define more carefully what criterion is being used to call the second harmonic a "phonon laser" during the squeezing protocol. An actual clear definition of phonon laser is missing in the text (see comment below). It would help to include linewidths, phase diffusion, power spectra before and after the pulse, or a more systematic characterization of coherence across the operating regime used for the headline squeezing values. Overall, the evidence that the system remains a coherent phonon laser throughout the squeezing protocol is not fully convincing.

Reply #4. We thank the reviewer for this critical and insightful comment. We fully agree that a clear definition of a phonon laser and a rigorous characterization of coherence throughout the squeezing protocol are essential to support our claims.

1. We define a phonon laser by three standard criteria [Refs. 12, 13]. These include a threshold condition for lasing, narrowing of the linewidth, and coherent oscillations quantified by the normalized correlation function $g^{(2)}(0)$ (at zero-time delay). In Supplementary Section 2 (Figure S1), we show that both the fundamental mode and the second harmonic meet these criteria. Notably, the amplitude of the second harmonic in the power spectral density (PSD in Figure S2) is observed to be lower (–12.2 dB) than that of the fundamental mode. Consequently, under identical measurement conditions, the second harmonic exhibits a reduced signal-to-noise ratio, resulting in performance degradation. Its $g^2(0)$ is about 1.2. This is higher than the fundamental, but it still satisfies the criterion for coherent oscillation (Supplementary Section 3, line 84-93).

2. The reviewer has mentioned whether the system stays coherent during the squeezing pulse. We have two observations. The squeezing pulse only changes the trapping stiffness via modulating the intensity of the trapping beam (532 nm). It does not change the pump power or the dissipative coupling. Therefore, the system remains above the threshold. As shown in Supplementary Figure S3, $g^2(0)$ stays almost the same for both modes in our measurement. This means the modulation itself does not cause decoherence. Coherence only drops when the power ratio becomes too large (Supplementary Section 3, line 106-122).

3. We check the phase-space in the Maintext. Both modes show clear limit cycles before and after the pulse. This also confirms that the phonon lasing state is maintained.

We have added necessary experimental data in Supplementary Section 3 and improved the manuscript accordingly to clarify this point (Supplementary Section 3, line 84-93).

Comment #5. 3. The key observable is the ratio \sigma_t^2/\sigma_0^2, extracted from repeated pulse sequences and phase-space reconstruction. The Methods state that 2500–3000 pulses were used per parameter set, and that each setting was repeated 20 times with standard-deviation error bars. This is useful, but still insufficiently documented in the main text. The authors should explain in more detail how the specific phase is selected for the variance extraction, how the phase drift is handled from shot to shot, and whether the reported minimum corresponds to a single time bin, an average over a phase window, or a fit. It would also be important to quote uncertainties directly on the dB squeezing values, not only on the raw variance points. As it stands, the difference between a result marginally above 3 dB and one at the 3 dB level is not rigorously established.

Reply #5. We thank the reviewer for the careful reading and constructive comments. We have addressed each of your four points in detail below, and made corresponding revisions in the manuscript.

1. "How the specific phase is selected for the variance extraction?"

In our protocol, the specific phase for variance extraction is not arbitrarily chosen but is directly determined from the squeezing angle. The initial squeezing angle depends on the relative magnitude of frequency (Supplementary Section 2). After the pulse, this angle varies with time t. We reconstruct the phase-space trajectory from repeated pulses (as shown in Fig. 2a of the main text) and assume the oscillator motion follows $A\cos(\Omega_1 t)$, where A is the mean of $\sqrt{q_i^2+p_i^2}$. Then, for each time bin relative to the pulse, we compute the noise distribution along the direction perpendicular to the instantaneous trajectory.

The actual squeezing angle, i.e., the quadrature that exhibits reduced noise, is determined by the pulse width. Specifically, for the fundamental mode, when the pulse width $\tau = T_2/4$ (where $T_2 = 2\pi/\Omega_2$), the squeezing angle at the end of the pulse is $\pi/2$ ("the specific phase"), meaning the momentum quadrature is maximally squeezed.

After the pulse, the entire phase-space distribution rotates at frequency Ω_1, so the direction of minimum variance evolves as $\pi/2-\Omega_1 t$. We always use the fitted $A\cos(\Omega_1 t)$ as the reference for the oscillation. Consequently, the noise distribution we extract is automatically tracked with the rotating frame (more details in Supplementary Section 4). The reported maximum squeezing (e.g., 3.15±0.35dB) corresponds to the time bin where the variance along the momentum direction is minimized.

2. "How the phase drift from shot to shot is handled?"

This is a critical point. We fully agree that phase drift would degrade the observed squeezing. In our experiment, the squeezing protocol does not require phase-locking between the pulse and the oscillation. Instead, we apply the frequency-switching pulse repeatedly (2500–3000 times per parameter set) with the sampling rate of 1 MHz. Because the pulse unlocks with the mechanical oscillation, the initial phase of the phonon lasers before the pulse is random from shot to shot.

We acknowledge that frequency fluctuation of the phonon lasers limits the squeezing compared to the theoretical value. The phonon lasers frequency exhibits slow drift due to the environmental fluctuations (temperature, air pressure, etc.). This frequency drift causes a mismatch between the two squeezing events within a single pulse (i.e., the two non-adiabatic frequency jumps at the beginning and end of the pulse), preventing the two squeezing effects to add constructively. As a result, the experimentally achieved squeezing is lower than the ideal theoretical value ($10\log_{10}\Omega_1/\Omega_2$).

3. "Whether the reported minimum corresponds to a single time bin, an average over a phase window, or a fit?"

The reported minimum squeezing (3.15±0.35 dB) corresponds to a single time bin (i.e., a specific instant after the pulse). Specifically, for each pulse width τ, we reconstruct the phase-space trajectory at discrete time bins with a resolution of 1 μs (limited by the 1 MHz sampling rate). For each time bin, we compute the momentum noise variance from the ensemble of 2500–3000 shots. The minimum variance across time is then recorded for that τ. We repeat this measurement for 20 independent datasets and then average the minimum variance value. The reported 3.15 dB is the mean of these 20 minima, and the uncertainty (±0.35 dB) is the standard deviation.

4. "Quoting uncertainties directly on the dB squeezing values."

We thank the reviewer for emphasizing the importance of reporting uncertainties directly on the squeezing degree. The squeezing degree is

3.15±0.35 dB (mean ± standard deviation). We have updated the main text and Supplementary Information. We greatly appreciate the reviewer's rigorous approach to data reporting.

We believe these revisions fully address the reviewer's concerns. All changes have been highlighted in the revised manuscript.

Comment #6. 4. The phase-space distributions rely on estimating momentum by differentiating measured position traces. The authors note that the time resolution is 1 μs, much shorter than the oscillation period and momentum relaxation time, and therefore infer that momentum can be estimated reliably. That is a reasonable starting point, but numerical differentiation is notoriously noise-sensitive. The manuscript should provide a control analysis showing that the apparent squeezing is not distorted by derivative filtering, finite bandwidth, or detector noise. A calibration using synthetic signals or an independently validated estimator for the quadratures would make the result much more robust.

Reply #6. We thank the reviewer for raising this important technical point. Numerical differentiation is indeed sensitive to noise. We have performed control analyses in Supplementary Section 5. These verify that the observed squeezing is not an artefact of the data processing.

1. Calibration with unsqueezed reference data.

Under identical experimental conditions (same optical potential, same data acquisition and processing chain), we performed measurements without applying the frequency-switching pulses. That is, no non-adiabatic frequency shifts were introduced. In this case, the system resides in a phonon lasing state. No squeezing is expected in this state.

We applied the same numerical differentiation, bandpass filtering, and phase-space reconstruction procedures to these reference data. The results show that the momentum noise variance ratio σ_t^2/σ_0^2 remains close to 1 for all phases. Fluctuations are below 0.02 dB. No observable noise reduction or amplification is present. This control confirms that our processing chain does not artificially generate a squeezing signature.

2. Calibration with synthetic signals.

We further validated the method using a synthetic reference signal: a pure coherent oscillation with added Gaussian white noise (signal-to-noise ratio ≈ 3 dB). Applying the same numerical differentiation and reconstruction pipeline to this signal yielded a circular distribution in phase space, and the extracted momentum noise variance ratio remained close to 1 across all phases. This confirms the reliability of our estimator.

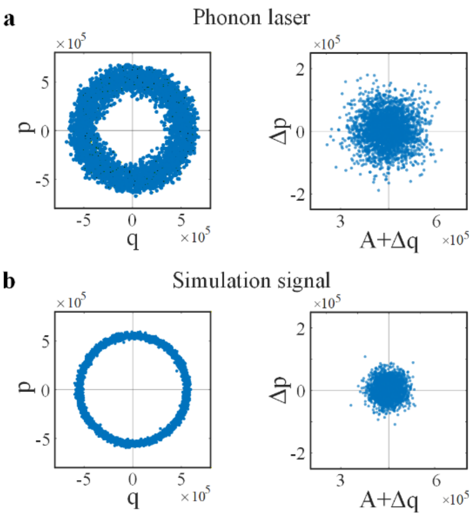

Figure 1. a. Phase-space distribution and quadrature noise of the phonon lasers. **b,** Phase-space distribution and quadrature noise of a synthetic reference signal. A pure coherent oscillation with added Gaussian white noise (SNR ≈ 3 dB).

Taken together, these controls demonstrate that the observed classical squeezing is a genuine physical effect and not an artefact of numerical differentiation, finite bandwidth, or detector noise. We have added this control analysis to the Supplementary Information Section 5 (line 207-234) in the revised manuscript.

Comment #7. 5. The theoretical description is too compressed and, in places, unclear. The non-adiabatic frequency-jump protocol is familiar, but here it is being applied to a self-oscillating nonlinear phonon-laser state. The manuscript jumps rather quickly from the switching of trap power to a harmonic-oscillator squeezing operator and a simple formula for the dB reduction. The authors should better justify the effective Hamiltonian used during the jump, state the assumptions under which the standard squeezing transformation applies, and explain how the harmonic component inherits squeezing from the same protocol. Eq. (5), which relates g_2 to quadrature variables, is also difficult to parse in its present form and should be rewritten more carefully.

Reply #7. We thank the reviewer for this important critique.

1. We agree that the theoretical presentation in the original manuscript was overly compressed, and we have substantially revised it to improve clarity and rigor. Below we detail our revisions.

The non-adiabatic frequency jump protocol is indeed a well-established method for generating squeezed states of a harmonic oscillator, with a rich history dating back to Janszky & Yushin (1986) and rigorously analyzed by Agarwal & Arun Kumar (*Phys. Rev. Lett.* 1991). Importantly, as shown by Agarwal & Arun Kumar, the frequency-jump protocol produces squeezing regardless of the initial state—whether it is a vacuum state, a coherent state, a thermal state, or even a squeezed state.

In our scheme, the Hamiltonian before and after the frequency jump is H_1 and H_2, respectively. When the non-adiabatic condition is satisfied (i.e., the switching time is much shorter than the mechanical period), the wave function of the mechanical oscillator does not have time to adapt to the new potential. Consequently, during the abrupt change from H_1 to H_2, one quadrature is squeezed while the orthogonal one is anti-squeezed. This process is equivalent to a Bogoliubov transformation of the quadrature operators:

$$\hat{b}_2^\dagger = \cosh(r)\hat{b}_1^\dagger + \sinh(r)\hat{b}_1 e^{i\theta} = \hat{S}(r)\hat{b}_1^\dagger \hat{S}^\dagger(r),$$
$$\hat{b}_2 = \cosh(r)\hat{b}_1 + \sinh(r)\hat{b}_1^\dagger e^{i\theta} = \hat{S}^\dagger(r)\hat{b}_1 \hat{S}(r),$$

where $\hat{S}(\xi) = \exp[(\xi^* \hat{b}_1^2 - \xi \hat{b}_1^{\dagger 2})/2]$ is the squeezing operator, $\xi = re^{i\theta}$ denotes the squeezing parameter, $r = \ln(\Omega_1/\Omega_2)/2$ (θ) is the squeezing magnitude (angle), and $\hat{b}_1(\hat{b}_2)$ is the annihilation operator before (after) the jump. In principle, the maximum achievable squeezing depends solely on the frequency ratio, with no reliance on external feedback or cooling.

Importantly, the same frequency jump can also be applied to the second-harmonic mode. In our nonlinear phonon lasers, the second-harmonic frequency is locked to $2\Omega_1$. A non-adiabatic change of the fundamental from Ω_1 to Ω_2 forces the second harmonic from $2\Omega_1$ to $2\Omega_2$. The frequency ratio $2\Omega_1/2\Omega_2 = \Omega_1/\Omega_2$ is the same as that of the fundamental mode, so the squeezing parameter $r = \ln(\Omega_1/\Omega_2)/2$ is identical for both modes.

2. Revision of Eq. (5) and the definition of $g^{(2)}(0)$:

$$g^{(2)}(0) = \frac{\langle \hat{b}^\dagger \hat{b}^\dagger \hat{b}\hat{b} \rangle}{\langle \hat{b}^\dagger \hat{b} \rangle^2} = \frac{\langle \hat{N}^2 \rangle - \langle \hat{N} \rangle}{\langle \hat{N} \rangle^2}.$$

where $\hat{N} = \hat{b}^\dagger \hat{b}$ denotes the phonon number operator and $\hat{b}\left(\hat{b}^\dagger\right)$ is the creation (annihilation) operator. The phonon number can be obtained from the view of the energy

$$\hat{H} = \frac{1}{2}\hbar\Omega_m\left(\Delta\hat{x}^2 + \Delta\hat{p}^2\right) = \hbar\Omega_m\left(\hat{b}^\dagger\hat{b} + \frac{1}{2}\right) = \hbar\Omega_m\left(\hat{N} + \frac{1}{2}\right),$$

where $\left(\hat{b}+\hat{b}^{\dagger}\right)/\sqrt{2}=\Delta\hat{x}=\hat{x}-\langle\hat{x}\rangle$, $\left(\hat{b}-\hat{b}^{\dagger}\right)/i\sqrt{2}=\Delta\hat{p}=\hat{p}-\langle\hat{p}\rangle$, $\hat{p}(\hat{q})$ denotes the dimensionless position (momentum) operator. With respect to the quadrature variables, the phonon number can be calculated by

$$\hat{N}=\frac{1}{2}\left(\Delta\hat{x}^{2}+\Delta\hat{p}^{2}-1\right).$$

In the revised manuscript, we have improved the theoretical section. These revisions significantly improve the clarity and rigor of the theoretical description (page 13-14, line 330-342 in the Maintext and Section 2 in Supplementary Information).

Comment #8. 6. There are already important prior demonstrations of squeezing in levitated optomechanics, including thermal and quantum squeezing via fast frequency changes, and the authors themselves previously reported nonlinear multi-frequency phonon lasers. The true novelty here appears to be the application of such squeezing to a self-oscillating phonon-lasing state, plus the observation of a squeezed second harmonic. That is interesting, but the manuscript should articulate this more precisely and compare directly with the closest prior works: the novelty relative to prior work needs to be sharpened. Right now, the introduction and discussion sometimes blur together squeezing of levitated oscillators in general and squeezing of a phonon laser in particular. A more focused positioning would strengthen the paper.

Reply #8. We thank the reviewer for noting the novelty of our work and for the valuable suggestion to sharpen the comparison with prior studies. In response to this and related comments from other reviewers, we have thoroughly revised the manuscript to better position our contribution.

1. In the Introduction, we have now explicitly distinguished between squeezing of a thermal levitated oscillator and squeezing of a phonon laser. Specifically, we cite the representative work on thermal squeezing (e.g., Rashid et al., *Phys. Rev. Lett.* 117, 273601 (2016) – 2.7 dB) and the subsequent study that used the same platform to characterize Duffing nonlinearities (Muffato, R. *et al*., *Phys. Rev. Res.* **7**, 013171 (2025).). Our revised text (page 2, line 45-49 in the Maintext) now reads:

"A classical squeezing of 2.7 dB has been achieved in a thermal nanosphere. Further squeezing in the same system has demonstrated a frontier exploration of inherent Duffing nonlinearities. However, squeezing a levitated sphere in the multimode phonon lasing state remains unexplored, and even more difficult for larger spheres due to low oscillating frequencies."

Compared with these earlier thermal squeezing works, which squeezed a thermal state of a nanosphere, our work performs classical squeezing on a phonon-lasing state with a microsphere. We have removed any ambiguous comparison with quantum squeezing experiments (e.g., cooled nanospheres) to avoid misinterpretation, and instead focus on the contrast between passive thermal oscillators and phonon lasers.

2. Regarding the reviewer's comment on the squeezed second harmonic, we agree that this point needs to be articulated more precisely. We not only squeeze the fundamental mode but also observe simultaneous squeezing of the second harmonic. It has not been reported in previous experiment. This is now stressed in the manuscript (page 2, line 50-52, and page 10, line 235-241 in the Maintext).

Besides, during the revision period, we became aware of a very recent study (Zhang, K. *et al*., *Nat. Commun.* **17**, 2882 (2026)) that reports two-mode thermal squeezing in a phonon laser. Our work instead shows single-mode squeezing of both the fundamental and second harmonic modes of a nonlinear phonon laser with a larger microsphere. The two results are complementary. To provide a fair and clear positioning, we have added a short paragraph in the Discussion section (page 11, lines 242–248) that states:

"Note added in proof: When submitting our revised manuscript, an experiment appeared, and in that work, a two-mode thermomechanically squeezed phonon laser has been demonstrated with a levitated nanoparticle in high vacuum[64]. Our work instead shows single-mode squeezing of both the fundamental and second harmonic modes of a nonlinear phonon laser with a larger microsphere. The two results are complementary, setting the stage to research of non-equilibrium physics and applications in precision metrology."

We believe these revisions address the reviewer's concern and sharpen the novelty of our work. Thank you again for the constructive feedback.

Style comment:

Comment #9. -) The manuscript lacks a broad audience introduction about phonon laser. The introduction is a mere list of previous works and possible applications. This is not conformal to the style of Nature Communications.

Reply #9. This professional opinion of the reviewer has been very helpful to us. We have substantially revised the introduction to better serve a broad audience. The revised introduction now clearly explains what a phonon laser is (threshold, linewidth narrowing, autocorrelation squeezing) and how squeezing enhances its performance by reducing the momentum (position) noise, which directly benefits multimode sensing and can be applied to the non-equilibrium thermodynamics. This makes the motivation and significance of our work more accessible (page 2, line 28-34, 38-40 in the Maintext).

Comment #10. -) Several claims about applications are overstated. The concluding discussion mentions macroscopic superposition, gravitational-wave detection, and broadly enhanced precision metrology. These (over)claims are not really supported by the present data. The experiment is performed at room temperature and 20 mbar, with squeezing persisting for only a few ms for the fundamental mode and less than 1 ms for the second harmonic. This is a respectable proof of principle, but it is still far from the mentioned applications. The discussion should be toned down and kept closer to what is actually demonstrated and realistic in the near future.

Reply #10. We thank the reviewer for this constructive comment. We agree that the original discussion contained overstated claims about applications such as macroscopic superposition and gravitational wave detection, which are not supported by the current data. As noted in our previous response, we have removed these overreaching statements. We now focus on the demonstrated advantages of our squeezed phonon lasers system in the classical domain, such as enhancing frequency discrimination for force and mass sensing, and providing a multi-frequency squeezed source for parallel detection. Moreover, our results can be applied to the multimode sensing and non-equilibrium thermodynamics. The corresponding revisions have been made in the conclusion section, where we now focus on the demonstrated advantages of our squeezed phonon lasers system in the classical domain (page 10-11, line 224-240 in the Maintext).

Minor comments.

Comment #11. -) The manuscript requires editing. There are many awkward or ungrammatical phrases, for example “more exacerbated,” “uses in a huge phonon number,” “coherence fails to retain,” and several figure-caption sentences that are syntactically broken. The adjective “huge” is used several time and it sounds non-scientific (huge with respect to which physical quantity?)

Reply #11. We respect the rigorous attitude of the reviewer. We have thoroughly revised the text to improve the grammar and style. Specifically:

1.The phrase “more exacerbated” has been replaced with “even more difficult”.

2.The phrase "uses in a huge phonon number" has been revised to "operates with a large phonon number".

3.The phrase "coherence fails to retain" has been corrected to "coherence is not retained".

4.The adjective "huge" has been replaced with more precise descriptors such as "large" or "high" throughout the manuscript.

5.We have also corrected several syntactically broken sentences in the figure captions.

Comment #12. -) The statement that the protocol is "inherently free from the 3 dB squeezing limit" is too quick and needs a clearer theoretical basis, especially since the reported value only marginally exceeds 3 dB.

Reply #12. We value every piece of feedback from the reviewer. The statement that the protocol is "inherently free from the 3 dB squeezing limit" was intended to contrast our method with conventional parametric amplification schemes. As explained in [*New J. Phys.* **17** 063018 (2015)], the instability threshold occurs because the dissipation of the amplified quadrature goes to 0. At the same time, the dissipation of the squeezed quadrature goes to twice its bare value. This causes a reduction in the variance of the squeezed quadrature by a factor of two, which corresponds to the 3dB limit.

In contrast, our squeezing protocol relies on a non-adiabatic frequency jump, which is a transient operation rather than a steady-state parametric process. Under such conditions, the squeezing is determined solely by the ratio of the initial and final oscillation frequencies, as given by Eq. (2) in the manuscript. This mechanism does not involve a dissipative balance and therefore does not encounter the same 3 dB limitation. Theoretically, there is no 3 dB limit in our protocol (see Supplementary Section 2), and a larger frequency ratio can yield squeezing beyond 3 dB without requiring additional feedback or pulsed parametric driving [*Phys. Rev. Lett.* **127**, 183602 (2021)].

We acknowledge that the experimental squeezing we achieved (3.15 ± 0.35 dB) only marginally exceeds the 3 dB limit, and our original phrasing may have implied a more definitive demonstration of beating this limit. To avoid overstatement, we have revised the text to clarify the theoretical distinction while toning down the claim. The revised sentence now reads:

"Through non-adiabatic frequency shifts induced by a pulse-modulated trapping laser, the fundamental-mode phonon laser was squeezed ***by*** *3.15 ±0.35 dB."* (page 1, line 20 in the Maintext)

Comment #13. -) The manuscript should clarify whether the "simultaneous" squeezing of the fundamental and second-harmonic modes means truly simultaneous joint observation in the same run, or simply that the same protocol can be applied to both modes under the same platform. The present figures show separate analyses rather than an explicitly joint measurement.

Reply #13. Thank you for your question. The term "simultaneous" in the manuscript refers to the fact that both the fundamental and second-harmonic modes are generated concurrently in the same experimental run, and the non-adiabatic frequency jump affects both modes at the same time. As a result, both modes experience squeezing simultaneously.

However, the optimal pulse duration for achieving maximum squeezing differs between the two modes due to their distinct oscillation frequencies. Specifically, the fundamental mode reaches its maximum squeezing at a pulse width of $T_2/4$ while the second-harmonic mode achieves optimal squeezing at $T_2/8$ (or $3T_2/8$). Therefore, although both modes are squeezed in the same run, the reported maximum squeezing values for each mode are obtained under separate optimized conditions. The figures (Fig. 2b, Fig. 3b) present the corresponding analyses accordingly.

To avoid confusion, we have revised the text to clarify this point. The revised statement now reads *"both the fundamental and second-harmonic modes are squeezed simultaneously in the same measurement, while their optimal squeezing occurs at different pulse width by comparing Fig. 2b and 3c."* (page 9, line 192-195 in the Maintext).

Comment #14. -) Maybe it would help the reader to include one panel with representative raw time traces before and after the squeezing pulse, not only reconstructed phase-space clouds.

Reply #14. We thank the reviewer for this helpful suggestion. To make the squeezing effect more intuitive, the method of data processing and figure presentation follow the approach used in a previous study on thermal squeezing of a levitated sphere [Rashid et al., *Phys. Rev. Lett.* **117**, 273601 (2016)]. Following the reviewer's suggestion, representative raw time traces before and after the squeezing pulse, together with the power spectral density (PSD), have been included in Supplementary Section 4 (page 8-9, line 141-155).

Comment #15. In summary, this is a promising experimental study, but the current manuscript overstates its implications and does not yet provide a sufficiently rigorous quantitative analysis of the squeezing measurement and its interpretation. I would therefore recommend major revision. With a clearer distinction between classical and quantum squeezing, stronger validation of the variance extraction and coherence claims, improved theory exposition, and a more restrained discussion of applications, the manuscript could become suitable for publication.

Reply #15. We sincerely thank the reviewer for the thoughtful and constructive feedback. We have carefully revised the manuscript to address all the raised points. Specifically, regarding the overstatement of implications, we have removed all speculative claims about macroscopic superposition, gravitational-wave detection, and the quantum-classical boundary, and refocused the discussion on the demonstrated advantages of classical squeezing in phonon lasers for force sensing, nonequilibrium dynamics, and multimode sensing.

Regarding the need for a more rigorous quantitative analysis of the squeezing measurement and its interpretation, we have now provided a clear definition of the noise variances (σ_t^2 and σ_0^2), quoted uncertainties directly on the dB squeezing values (e.g., 3.15±0.35 dB), and included control analyses using synthetic signals and unsqueezed reference data to validate our phase-space reconstruction and differentiation method. We have also improved the theoretical exposition and clarified the distinction between classical and quantum squeezing throughout the manuscript. We believe these revisions have significantly strengthened the work.

Responses to the comments of Reviewer #3

We appreciate Reviewer #3's thorough reading and constructive feedback. We have carefully revised the manuscript accordingly, with all changes clearly marked in red.

Comment #1. The manuscript by Song et al. reports the realization of squeezed phonon lasers. It shows modest (up to ~3 dB) squeezing of the thermal noise of a phonon laser realized in an active optomechanical cavity. While the experimental setup and its ability to realize phonon lasing was described in earlier work (Ref. 15), the novelty of the current manuscript lies in the fact that the thermal noise of this multimode phonon lasing platform with a large mass is now being squeezed, and in fact in two mechanical modes simultaneously.

Reply #1. We are grateful to the reviewer for acknowledging the novelty of our work. As noted, the experimental platform builds on our earlier work (Ref. 13), but the present manuscript extends its functionality by demonstrating classical squeezing of the multimode phonon lasers. Specifically, by applying non-adiabatic frequency jumps, we achieve classical squeezing of both the fundamental mode and its second harmonic without any feedback control.

Comment #2. I find the work generally to be of interest, and recognize a degree of novelty for the above reasons. Unfortunately, the manuscript does not explain however what the value is of squeezed phonon lasers (either over squeezed thermal or vacuum states or over coherent-state phonon lasers), and thus how this constitutes a significant advance that warrants dissemination to a broad readership. The introduction and conclusions are vague in describing the benefits of squeezed lasing, going no further than "squeezing in phonon lasers is significant because it could reduce thermal noise and increase coherence" and "the thermal noise of the nonlinear phonon lasers is suppressed effectively". Such descriptions are too simple and factually incorrect: Squeezing reduces the fluctuations in one degree of freedom at the expense of increasing the fluctuations in another degree of freedom. As such, one cannot say that it "reduces thermal noise", and it is also not clear what specific applications benefit from thermally squeezed phonon lasing, in comparison to e.g. coherent phonon lasing with reduced fluctuations in both quadratures or the original lasing state. This complete lack of description of significance and prospect make the manuscript in its current form unsuitable for Nature Communications.

Reply #2. Thank you for the thorough review and constructive suggestions. We agree that the original manuscript did not clearly articulate the specific value of squeezed phonon lasers. To address this, we have revised the introduction and conclusion as follows.

1. In phonon-laser-based sensing applications, an external weak force typically induces a frequency shift of the phonon lasers (Knünz, S. et al. *Phys. Rev. Lett.* **105**, 013004 (2010), Pan, F. et al. *Chip* **2**, 100050 (2023)). The resolution of such a sensor is therefore limited by the phase stability (or frequency fluctuations) of the lasers. Squeezing the phonon lasers can reduce noise in one quadrature while increasing it in the orthogonal quadrature. The reduction of the noise is a well-known advantage of squeezing in metrology and has been extensively studied in optical lasers, while our work demonstrates it for the first time in a nonlinear phonon laser (page 2, line 38-40 in the Maintext).

Furthermore, as emphasized in our revised conclusion (page 11, line 235-241 in the Maintext), the simultaneous squeezing of the fundamental and second harmonic modes provides a multi-frequency squeezed source. This offers valuable resources for the researching in nonequilibrium dynamics, nonlinear phononics, and multimode sensing.

2. Additionally, we have corrected some inappropriate expressions in the manuscript. We have removed the phrase "reduce thermal noise" and replaced it with "suppressing momentum (position) noise at the cost of increased noise in the orthogonal quadrature" (page 2, line 38-39 in the Maintext). We believe these clarifications help establish the significance of our work for a broad readership.

3. Actually, as the reviewer mentioned, there exists other techniques, such as injection locking, feedback control, and nonlinear cooling, that can simultaneously reduce fluctuations in both quadratures. Our group has already employed injection locking in the active levitated optomechanical system, which significantly improves the brightness and stability of the nonlinear phonon lasers (Xiao et al., *eLight* **4**, 17 (2024)). These methods are indeed valuable for improving phonon lasers performance and have been demonstrated in various systems. Squeezing offers a complementary approach. Squeezing can further suppress the noise in a chosen quadrature, at the expense of increasing the noise in the orthogonal quadrature. In such a way, it can be applied on top of other noise-reduction techniques to push the performance beyond what is achievable by either method alone. We have added a paragraph in the Discussion section (page 10, line 224-234).

Comment #3. Moreover, the manuscript is unclear in distinguishing between quantum squeezing and (classical) thermal squeezing. The reported experiments concern the latter and are far from the quantum squeezing regime. Still, in the introduction this distinction is completely ignored when comparing to other work on squeezed mechanical states (including true quantum squeezing). This is inappropriate, and does not do justice to the efforts of others to achieve significant quantum squeezing, which is arguably a much more challenging task. (The authors suggest that squeezing is more impressive with a lot of thermal noise, but that is incorrect.) This manuscript should explicitly refer to the observations as thermal squeezing, in the title, the abstract, the introduction, and the conclusions, since the single word 'squeezing' is too ambiguous without proper context and explanation. Moreover, the statement "It provides a powerful capability for probing the fundamental quantum-classical boundary." should be removed, since it does not make sense in light of the above. In fact, I see no impact of this work on quantum science in its current form.

Reply #3. We sincerely thank the reviewer for this thorough and insightful critique. We fully agree with your assessment that our manuscript did not sufficiently distinguish between quantum squeezing and classical (thermal) squeezing, and that this distinction is critical for the proper interpretation of our results. We have carefully revised the manuscript to address all your concerns, as detailed below.

We have followed the reviewer's suggestion and made the following explicit changes:

1.Abstract: Replaced "squeezing" with "classical squeezing" and "squeezed nonlinear phonon lasers" with "classically squeezed nonlinear phonon lasers".

2.Introduction: Clarified the comparison with prior quantum squeezing works (e.g., Kamba et al., *Science* 2025) to avoid any misrepresentation. The original statement regarding "squeezing is more impressive with a lot of thermal noise" (original manuscript page 2, line 46) was misleading. We have corrected it in the revised manuscript (page 2, line 45-47 in the Maintext).

3.Conclusions: We have thoroughly revised the conclusions to better highlight the relevance of our work to nonlinear phononics, non-equilibrium physics, and precision metrology (e.g., multimode sensing). Specifically, we have removed overreaching statements about the "quantum-classical boundary" and "gravitational-wave detection", and instead explicitly state that the observed squeezing is classical thermomechanical squeezing. We now connect our results to recent advances in phonon-laser-based sensing, including mass sensing and force detection, and explain how squeezing provides a complementary route to enhance the signal-to-noise ratio. We also discuss the future extension to squeezed phonon laser combs (page 10, line 224-234 in the Maintext).

In addition, I have several smaller questions and observations:

Comment #4. - In Figure 1a and Appendix Figure S1, the authors present the measured PSD, which appears to contain residual modes beyond the fundamental mode and the second harmonic. Could the authors comment on the origin and influence of these additional modes? They seem more pronounced at trapping power P_1, while they appear to be less prominent at P_2. How do these modes affect the reported results?

Reply #4. We thank the reviewer for this careful observation. We have analyzed the origin of these additional peaks in the power spectral density and their potential influence on our results.
As depicted in Fig.2, the residual peaks can be categorized into two types:

1. Modes from other spatial dimensions (*y* and *z* axes): The trapped microsphere possesses three translational degrees of freedom (*x, y, z*). In our experiment, the phonon lasing is primarily excited along the x-direction. Their fundamental frequencies and corresponding higher harmonics appear as additional peaks in the PSD. These peaks are indicated by the blue boxes in the revised Supplementary Figure S4.

2. Mechanical sidebands: When the system operates in the high-gain phonon lasing regime (especially at higher trapping power P_1), the strong nonlinear optomechanical coupling can generate sidebands around the fundamental and harmonic frequencies. These sidebands become more pronounced when the nonlinearity is strong, which are indicated by the red boxes in Figure 2.

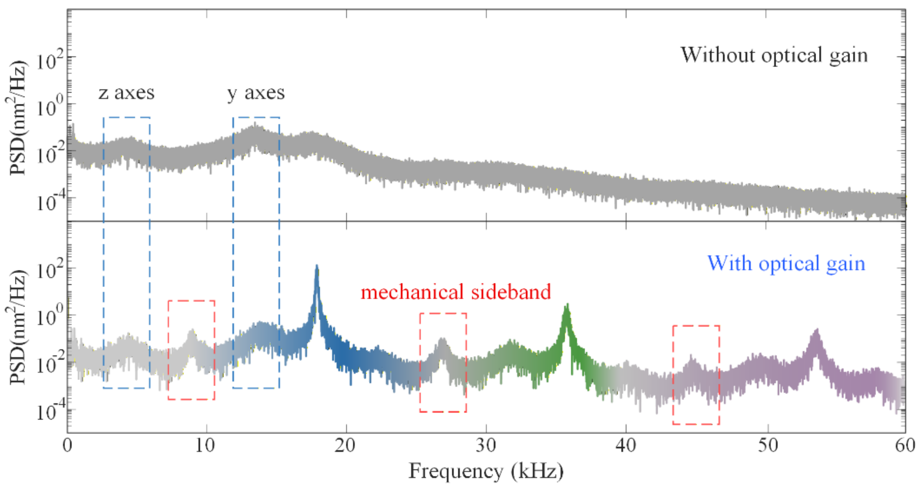

Figure 2. Power spectral density (PSD) of the trapped microsphere displacement. Top panel: Thermal state. Bottom panel: Phonon lasing. The blue boxes indicate residual peaks originating from the orthogonal y- and z-axis mechanical modes and their harmonics. The red boxes mark mechanical sidebands arising from strong nonlinear optomechanical coupling in the lasing regime.

At the lower trapping power P_2, the nonlinear interaction between the microsphere and the cavity field is weaker, which explains why these sidebands are less prominent or absent.

Meanwhile, we have carefully evaluated whether these residual modes affect our squeezing. The key point is that the amplitudes of these residual peaks are significantly lower (typically 3 orders) than those of the fundamental mode and the second harmonic mode, which are the focus of our study. As we have detailed in the Supplementary Information (Section 4: Data processing), we apply a simple bandpass filter around the frequency of interest before reconstructing the phase-space distribution and calculating the quadrature variances. This filtering effectively removes the contribution of all out-of-band modes, including the y/z modes and the mechanical sidebands.

Therefore, the reported squeezing results are not affected by these residual modes. We have added a clarifying sentence in the revised Supplementary Section 3 (page 8-9, line 141-155) to explicitly state this point.

Comment #5. - In Figure 1d, the authors present the state before the squeezing pulse. From the small oscillations visible in the plot, it appears that the state may already be slightly squeezed prior to the application of the squeezing pulse. Could the authors comment on how their results would change if the initial state were a perfectly thermal state?

Reply #5. We thank the reviewer for this careful observation.

1. We confirm that the initial state before the squeezing pulse is not squeezed. Before the squeezing pulse, the mean ratio σ_t^2/σ_0^2 is 1.006±0.014 (σ_0^2 is the dimensionless momentum noise of the phonon lasers), where the slight offset is due to the measuring error and the fluctuation attributes to environment noise. In our experiment, a 5 ms interval was performed to make the squeezed phonon laser evolute to its initial state.

2. "How their results would change if the initial state were a perfectly thermal state?"

If the initial state were a perfectly thermal state, the same non-adiabatic frequency switching protocol would generate a thermal squeezed state. This is fundamentally similar to the classical squeezing in our phonon lasers. The underlying theory is the same: the squeezing degree depends only on the frequency ratio and the pulse duration, regardless of whether the initial state is thermal or coherent.

The main difference lies in the phase-space distribution. For a thermal state, the distribution remains Gaussian and centered at the origin, without a limit cycle. In contrast, our phonon lasers exhibit a limit cycle due to self-sustained coherent oscillation. We note that such thermomechanical squeezing of a thermal state has already been demonstrated in a levitated nanosphere at room temperature by Rashid et al. (*Phys. Rev. Lett.* **117**, 273601 (2016)). This work used the same frequency-jump technique and achieved up to 2.7 dB of squeezing.

Comment #6. - In Table S1, the authors note that the mass of the oscillator is approximately 10^(-14) kg. Could the authors comment on how the uncertainty in the oscillator mass propagates to the uncertainties in the p and q axes in Figures 1, 2, and 3?

Reply #6. We thank the reviewer for this important question regarding the role of mass calibration in our phase-space reconstruction. We analyze the propagation of mass uncertainty from the perspective of the uncertainty principle.

While the degree of squeezing is mass-independent, the numerical values of q and p plotted on the axes of Figs. 1-3 do depend on the assumed m. However, these axes are labeled in units of $\sqrt{\hbar/2m\Omega}$ and $\sqrt{\hbar m\Omega/2}$, which are derived units. If the true mass differs from our estimated value, the physical interpretation of the axis ticks changes, but the shape of the phase-space distribution (ellipticity, rotation, squeezing angle) remains unchanged because both coordinates scale by the same factor $\sqrt{m}$.

Therefore, our conclusions about the degree of squeezing are robust against mass miscalibration.

The certified microsphere (GBW12012b, SiO_2) has a measured value of 2.062±0.02 μm. Using the density of SiO_2 (ρ=2200kg/m^3), the mass is 1.03 ×10^{-14}kg. The relative uncertainty in the radius is $\Delta R/R \approx 0.97\%$, leading to a relative mass uncertainty:

$$\frac{\Delta m}{m} = 3\frac{\Delta R}{R} \approx 2.91\%.$$

Therefore, the relative uncertainty in q and p due to mass is:

$$\frac{\Delta q}{q} = \frac{\Delta p}{p} = \frac{1}{2}\frac{\Delta m}{m} \approx 1.45\%.$$

We have added a note in the Supplementary Section 5 (page 13, line 221-230) clarifying this point.

Comment #7. - In line 53, the authors report a maximum squeezing of 3.15 dB for the fundamental mode and 1.23 dB for the second harmonic. From Figures 2b and 3c, it appears that these degrees of squeezing are only achieved when using different pulse lengths for the two modes. Could the authors comment on the practicality of using two different pulse lengths if this protocol were applied in sensing or other applications? Additionally, could the authors comment on the maximum degree of squeezing that is achieved when using a single pulse length for both modes?

Reply #7. We thank the reviewer for this insightful question. Indeed, the optimal pulse duration for squeezing depends on the target mechanical frequency, as the non-adiabatic squeezing condition scales with the period of the oscillator. Therefore, using a single square pulse, it is possible to simultaneously squeeze both modes, but not to achieve the maximum squeezing for both at the same time.

To address the reviewer's question on the squeezing degree achievable with a single pulse length for both modes, we have performed additional analysis using the data presented in Figs. 2b and 3c. As an example, we take the pulse width that optimizes the second-harmonic mode, i.e., $\tau = T_2/8$. Under this condition, the second-harmonic mode reaches its maximum squeezing of 1.23 dB (as reported). The fundamental mode exhibits a reduced squeezing of approximately 1.67 dB.

We note the following practical considerations:

1. In many sensing scenarios, one is interested in a particular mechanical mode (e.g., the fundamental mode for force sensing, or a higher harmonic for specific nonlinear detection). In such cases, the pulse length can be tailored to the mode of interest, while other modes are not required. Therefore, the protocol remains directly useful for single-mode applications.

2. By combining other techniques such as Floquet engineering [ref.28,29], the intensity and coherence of higher harmonics can be significantly enhanced. In such an improved system, it may become possible to design a single pulse shape that simultaneously achieves near-optimal squeezing for multimode sensing (e.g., the 1st, 3rd, and 5th harmonics, or the 2nd, 4th, and 6th harmonics).

We have added a brief discussion of these points in the revised manuscript (page 11, line 235-241 in the Maintext) to clarify the practicality and future possibilities of the protocol.

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We are grateful to both reviewers for their thorough and helpful comments. Their suggestions have greatly improved both the quality and clarity of our work. In the following, we respond to each comment in detail, point by point.

Responses to the comments of Reviewer #2

I thank the authors for the detailed and careful reply. In my view, the authors have addressed the core points I raised and have substantially improved the paper. The main experimental claims are now sufficiently clear and supported by the data. The work reports the classical squeezing of a nonlinear phonon laser in a levitated optomechanical system, including squeezing of both the fundamental mode and the second-harmonic mode. The experiment is technically interesting, and the combination of phonon lasing, levitated optomechanics, non-adiabatic frequency modulation, and multimode nonlinear mechanical dynamics makes the result suitable for publication in Nature Communications.

We are truly grateful for your assessment and for recognizing that we have addressed the core issues and substantially strengthened the paper. We greatly appreciate your acknowledgement that the experimental claims are now clear and well supported, and that the combination of phonon lasing, levitated optomechanics, frequency modulation, and multimode nonlinear dynamics makes the work suitable for Nature Communications. Your constructive suggestions have been invaluable in guiding us to refine the presentation of both the data and the underlying physical picture. In this revision, we have responded to each of your comments point-by-point.

Comment #0. To recommend the publication to Nature Communications I need the authors to include the wording classical or thermomechanical squeezing explicitly in the title.

Reply #0. We thank the reviewer for this constructive and necessary suggestion. In response, we have explicitly added the word “Classically” to the title to accurately indicate the nature of the squeezing.

Besides that, I do not think that another round of external review is necessary. Nevertheless I have minor concerns to be addressed:

Comment #1. Clarify the novelty claim.

The manuscript still uses broad statements such as “the first experimental demonstration” of a squeezed phonon laser. Given the note added in proof and the existence of a closely related recent experiment on a thermomechanically squeezed phonon laser, the novelty claim should be made more precise throughout the abstract, introduction, and discussion. A more accurate formulation would emphasize the specific contribution of the present work: single-mode classical squeezing of a nonlinear phonon laser in a levitated microsphere, including squeezing of the second-harmonic mode and operation under low-vacuum conditions.

Reply #1. We fully agree with the reviewer's assessment. Indeed, during our revision, a closely related experiment on a two-mode classically squeezed phonon laser appeared, making it necessary to sharpen our novelty claims. In response, we have revised the Abstract by changing the broad "the first experimental demonstration" to "an experimental demonstration of a classically squeezed phonon laser" (line 19) to avoid overclaiming.

At the same time, we would like to point out that our manuscript contains explicit descriptions of our specific contribution, namely, single-mode classical squeezing of both the fundamental and second-harmonic modes of a nonlinear phonon laser under low-vacuum conditions. We have realized single-mode classical squeezing of a nonlinear phonon laser, including squeezing of the second-harmonic mode (lines 23-25 in the Abstract, lines 56-58 in the Introduction, and lines 229-231 in the Discussion, as well as in the Note added in proof). Moreover, we have emphasized that our squeezing is achieved under low-vacuum conditions (lines 25-26 in the Abstract, lines 58-60 in the third paragraph of the Introduction, and lines 226-228 in the Discussion).

Comment #2. Clarify the meaning of "simultaneous" multimode squeezing.
The authors state that the fundamental and second-harmonic modes are squeezed simultaneously. However, the optimal pulse widths for the two modes are different. The text should explicitly distinguish between "both modes are affected by the same modulation protocol" and "both modes reach their optimal squeezing at the same pulse width," which does not appear to be the case. If the authors wish to retain the word "simultaneous," they should provide a precise quantitative statement of the squeezing observed for both modes under the same pulse conditions.

Reply #2. Sincere thanks to the reviewer for the careful comment. We agree that "simultaneous" needed clarification. To address this, we have revised the text (lines 198–202 in the revised manuscript) to explicitly distinguish between "both modes are squeezed under the same modulation protocol" and "both modes reach their respective optimal squeezing under different pulse conditions."

"For example, at the pulse width of τ = 18 μs, we observe a squeezing of 1.67±0.21 dB for the fundamental mode and 1.23±0.17 dB for the second-harmonic mode, as shown in Figs. 2b and 3c. However, the optimal squeezing for each mode occurs at different pulse widths ($\tau = nT_2/4$ for the fundamental and $\tau = (2n-1)T_2/8$ for the second harmonic)."

This revision makes it clear that while both modes are indeed squeezed simultaneously under the same experimental sequence, their optimal squeezing conditions are different.

Comment #3. Check the equations and notation in the squeezing protocol.
Equation (1), the definition of the squeezing operator, and Eq. (2) should be checked carefully for typesetting, signs, and numerical factors. The relation between the frequency ratio, the squeezing parameter, and the squeezing expressed in dB should be written in a transparent way, especially because the experimental value is compared directly with the theoretical expectation. The notation involving Ω_1, Ω_2, $2\Omega_1$, T_1, and T_2 should also be made unambiguous in the discussion of the second harmonic.

Reply #3. We appreciate the reviewer's careful and rigorous assessment. In response, we have thoroughly reviewed and revised the relevant parts of the manuscript.

First, we have examined the squeezing operator in Eq. (1) and optimized the typesetting of variables and formulas throughout the relevant section to eliminate potential ambiguities. Regarding the relation between the frequency ratio and the squeezing degree, Eq. (2) now clearly shows how the theoretical squeezing in dB is determined by the frequency ratio Ω_1/Ω_2 via the squeezing parameter $r = \ln(\Omega_1/\Omega_2)/2$. The detailed derivation of this relation has been further expanded in the Supplementary Information (Supplementary Note 2).

Second, to clarify the notation for the second-harmonic mode, we would like to point out that in the theoretical part of the main text, Ω_1 and $2\Omega_1$ denote the fundamental and second-harmonic frequencies before modulation, while Ω_2 and $2\Omega_2$ denote the corresponding frequencies after modulation. To make this even more explicit and avoid any confusion for the reader, we have added the following sentence in the main text (line 169-171 in the Maintext): "*During the pulse modulation, the frequency of the second-harmonic mode jumps between $2\Omega_1$ and $2\Omega_2$, following the frequency switching of the fundamental mode.*" Accordingly, $T_2/8$ corresponds to a quarter period of the modulated second-harmonic mode ($2\Omega_2$).

Comment #4. Avoid using (g^{(2)}(0)) as evidence of squeezing.
The variance reduction is the evidence for squeezing. The role of (g^{(2)}(0)) is instead to show that the phonon-laser coherence is preserved, or at least monitored, during the modulation protocol. The corresponding wording should be revised.

Reply #4. We would like to thank the reviewer for this important clarification. We fully agree that variance reduction is the direct evidence of squeezing, while $g^{(2)}(0)$ should be used to monitor whether coherence is preserved during the modulation. In our manuscript, we use the measured ratio σ_t^2/σ_0^2 to evaluate the degree of squeezing (line 118-120). The sentence in lines 148-149 ("*$g^{(2)}(0)$ is nearly insensitive ($g^{(2)}(0) \approx 1$) to the pulse width τ, indicating that phonon coherence is preserved over the pulse duration.")* is precisely intended to show that coherence is maintained during the modulation, not as evidence of squeezing.

Comment #5. Moderate the statement about the 3 dB limit.
The claim that the method is free from the 3 dB limit should be phrased carefully. The measured fundamental-mode squeezing is only slightly above 3 dB within the stated uncertainty, while the second-harmonic squeezing is below 3 dB. The authors may state that the protocol is not fundamentally constrained by the usual steady-state 3 dB parametric-squeezing limit, but they should avoid implying that the experiment has conclusively demonstrated strong squeezing far beyond this limit.

Reply #5. We thank the reviewer for this careful and rigorous comment. We agree that the claim about the 3 dB limit should be moderated. While the non-adiabatic frequency modulation scheme is indeed free from the 3 dB limit, we acknowledge that our experimental data do not show a significant excess beyond 3 dB. To avoid any potential overstatement, we have removed the original sentence "*Another aspect is that this universal approach based on non-adiabatic frequency shifts is inherently free from the 3 dB squeezing limit without the need of any complicated external feedback control techniques*" from the manuscript (lines 173–175).

Comment #6. Language and typographical corrections.
The manuscript still contains several grammatical and typographical issues. Examples include awkward phrases such as "a squeezing and anti-squeezing processes," "a high degree of squeezing occurs," "different pulse width," and the typo "unqueened phonon lasers." These should be corrected before publication.

Reply #6. We sincerely appreciate the reviewer's efforts in identifying the language and typographical issues in our manuscript. We have corrected

the following errors:
1. "undergoes a squeezing and anti-squeezing processes" → revised to "undergoes squeezing and anti-squeezing" (line 124)
2. "unqueened" → revised to "unsqueezed" (line 239)
3. "different pulse width" → revised to "different pulse widths" (line 201)
4. "The minimum value changes with the normalized variance varies with the pulse width" → revised to "The minimum value of the normalized variance varies with the pulse width τ" (line 162–163)
5. "a deeper squeezing" → revised to "stronger squeezing" (line 192)
6. "A high degree of squeezing occurs at pulse widths of $T_2/8$ and $3T_2/8$, respectively." has been revised to "Two minimum values appear at pulse widths of $T_2/8$ and $3T_2/8$." (line 185-186)

We are grateful to the reviewer for pointing out these issues, which have improved the overall quality of our presentation.

Overall, I find the revised manuscript scientifically sound and timely. The remaining issues concern precision of presentation rather than the validity of the main result. I recommend acceptance after the above minor revisions and, above all, revision of the the title (I do not need to see this manuscript again).

We are truly grateful to the reviewer for the positive overall assessment and the recommendation for acceptance. We have revised the title to include "classically" as suggested, and we have carefully addressed all the remaining presentation issues noted by the reviewer. We sincerely appreciate the reviewer's time, expertise, and constructive feedback throughout this process.

Reviewer #3:

The authors have substantially revised their manuscript, removing the problematic statements on impact and signficance, and clarified the fact that their work is in the classical domain. They have also addressed my technical questions. I remain not fully convinced that the demonstrated squeezing can have an actual benefit to sensing applications, since I believe it may effectively reduce interesting signals by similar degree as the thermal noise that is squeezed. That said, I understand that that is an open debate and the authors are not the only ones making such arguments. This paper does contribute to the body of work on squeezing in classical resonators, in this case a phonon laser. It could thus indeed be of sufficient significance for publication in Nature Communications. My earlier objections were taken away by the removal of the over-selling arguments.

We thank the reviewer for the positive assessment and the recommendation for acceptance. We appreciate the reviewer's thoughtful comment regarding the practical benefit of squeezing for sensing applications.

We acknowledge that this is indeed an important and open question. Here, we would like to offer a brief clarification. The basic principle of squeezing-enhanced sensing is straightforward: reducing the noise in one quadrature of the oscillator directly lowers the detection limit along that quadrature. This is the same strategy that has been successfully employed in gravitational-wave detection using squeezed light. In the mechanical domain, recent theoretical work has predicted that squeezed states can enhance force sensitivity by at least one order of magnitude (Y.-F. Qiao et al., *Phys. Rev. Lett.* 136, 040801 (2026)), and experiments on trapped-ion force sensors have already demonstrated that applying 3 dB of squeezing enables the detection of forces as small as ~86 yN (Z. Liu et al., *Phys. Rev. Appl.* 16, 044007 (2021)). These examples suggest that the benefit of squeezing is not merely speculative but has been experimentally validated in related systems.

That said, we agree with the reviewer that the quantitative benefit for a specific sensing task depends on the measurement scheme and the noise background. We present our results as a contribution to the broader body of work on classical squeezing in mechanical resonators, rather than making overreaching statements about immediate practical applications. We are grateful for the reviewer's balanced perspective and for recognizing the significance of our work.