nature portfolio

Peer Review File

Observation of Restored Adiabatic State Transfer in Time-Modulated Non-Hermitian Systems

Corresponding Author: Professor Peng Xue

This file contains all reviewer reports in order by version, followed by all author rebuttals in order by version.

Attachments originally included by the reviewers as part of their assessment can be found at the end of this file.

Version 0:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
The manuscript reports an experimental realization of adiabatic and symmetric state transfer in a time-modulated non-Hermitian two-level system by dynamically encircling an exceptional point. The authors show that non-adiabatic transitions can be suppressed when the evolution operator exhibits a purely real spectrum, leading to direction-independent eigenstate exchange. The topic is timely and relevant within the context of non-Hermitian physics, and the experimental implementation appears technically sophisticated, extending previous studies on chiral mode switching associated with complex spectra. At the same time, the current presentation may leave room for ambiguity in the interpretation of the reported state exchange.

In the manuscript, the Hamiltonian depends on a two-dimensional set of control parameters (x,y), and the eigenenergies discussed correspond to instantaneous values evaluated along this parameter dependence. The use of the term "spectrum" in this context therefore refers to these parameter-dependent eigenvalues, but its relation to the conventional notion of a spectral structure is not explicitly specified.

This issue is particularly evident in Fig. 1(a), where a line-like feature appears that is visually reminiscent of a branch cut in the conventional sense associated with complex multivalued functions. When viewed strictly within a two-dimensional parameter-space picture, such a structure could be taken to suggest an extended real-energy degeneracy, which would prevent adiabatic following. By contrast, branch cuts associated with complex eigenenergies do not necessarily preclude adiabatic evolution. At the same time, this feature may arise from how higher-dimensional parameter dependence is represented in two dimensions, from conventions used to label eigenstates, or from how continuity is enforced in the chosen representation. Clarifying the physical meaning of this branch-cut–like structure, whether it reflects an intrinsic spectral property or instead results from representational choices, is therefore important for how the figure is understood.

A closely related point arises in the interpretation of the dynamical results shown in Fig. 2. There, an abrupt change in the identified eigenstate occurs at a location that coincides with the branch-cut–like structure in the two-dimensional representation. Such behavior may be understood as resulting from how instantaneous eigenstates are defined or tracked within the chosen framework, rather than from a genuine spectral singularity encountered during the evolution. If this distinction is not clearly stated, the observed transition may appear to be directly associated with a spectral gap closing, even though the underlying mechanism may be different.

The Supplementary Material introduces a description in a four-dimensional parameter space, in which features that appear singular in the two-dimensional projection correspond to smooth structures with a finite complex spectral gap, except at isolated exceptional points. This higher-dimensional perspective provides useful context for interpreting the two-dimensional figures and for understanding how adiabatic evolution is restored in the present system. However, the role of this description in relation to the main text is not entirely clear.

These points are particularly relevant given that the experimental protocol implements time evolution in a discretized, stepwise manner, where the interpretation of adiabaticity is tied to the instantaneous spectral structure along the driven path. When features appearing in reduced or projected representations may reflect parametrization or visualization choices rather than intrinsic spectral properties, this distinction becomes important for interpreting the observed state exchange. As an

optional point, not essential to the main claims, examining how the behavior varies with the driving or modulation rate could help further place the experiment within an adiabatic context and may also provide indirect quantitative information about the effective minimum spectral gap along the trajectory, even when this gap is not directly visible in the two-dimensional representation.

Overall, the manuscript addresses an appealing physical idea and presents technically impressive experimental results. However, in its current form, the distinction between representational features and intrinsic spectral properties is not always evident, particularly in connection with the use of the term "spectrum," the appearance of branch-cut–like structures, and the interpretation of adiabaticity. Making these distinctions more transparent would strengthen the conceptual clarity of the work and reduce the risk of misinterpretation.

Reviewer #2

(Remarks to the Author)
Report for 'Observation of Restored Adiabatic State Transfer in Time-Modulated Non-Hermitian Systems' by Xiaowei Wang, Ievgen I. Arkhipov, Quan Lin, Huixia Gao, Dengke Qu, Lei Xiao, Franco Nori, and Peng Xue (NCOMMS-25-90175).

The authors present in their manuscript an optical experimental realization to encircle an EP but keeping an adiabatic state transfer which is based on a theoretical work [33]. The basic idea is to encircle the EP on a purely real Riemanian sheet. They find a good agreement between their experimental results and the theoretical predictions and also show that in case of using a complex sheet the non-adiabatic behaviour is found.

I find the paper interesting, well written, and think deserves to be published in Nature Communications if two issues can be explained:

1) Is the experiment really encircling the exceptional point dynamically (I can see this in the setup of the appendix, p4) but from the experimental side I have the impression that its just two lines (one polarization going through VOA, the other through DL) and they interfere via the PBC. Thus at the moment for me the experiment is mimicking the results of an EP encircling and not really performing the encircling! Maybe to address this question a reduced scheme in fig 1 will be helpful just detailing the principle leaving out BPF, EDFA, VPC etc.. This will be helpful for readers not coming from optics anyway. Also it might be helpful what are the experimental parameters that are varied (ratios etc). How many experiments have been performed for a single point? Why are the error bars only from the polarimeter, is it experimental or the precision of the device?
To be very precise how are the N=5x10^4 U(t) are applied for the wt=2pi measurement. Where is the feeding in of the U_n outgoing signal to the U_(n+1) ingoing signal. If I understand correctly the VOA is applying a single U_n?

2) I'm missing a detailed discussion on the effects of the linearization and discretization of the time steps.
This implies that the trajectory is not perfectly sitting on the real Riemannsheets but slightly go into the complex parts.
The theoretical discussion to perform this adiabatic state transfer around the EP is based on the fact that one stays "always" on the real sheets. Thus it might be more related on how fast the adiabatic process is.
There have been a paper which displays this (also experimentally):
PHYSICAL REVIEW A 102, 040201(R) (2020)
Encircling exceptional points as a non-Hermitian extension of rapid adiabatic passage
DOI: 10.1103/PhysRevA.102.040201
Could you please detail the difference of this work and the rapid adiabatic passage scheme.
In any case the paper should be cited and discussed in the introduction as it displays the same result as the authors are aiming for in this manuscript.

The replies to this two issues are crucial to see whether its novelty is sufficient to be published in nature communications.

Detailed remarks:

1) The statement "ultra-high-precision sensing [12–14]," should be made with care as nowadays it is more or less agreed that due to noise enhancement this ultra high sensing is not possible (thought sensors based on EPs might still be useful/partially outperforming other)

2) I think the equations for epsilon, ..., kappa should be given an equations number and the relations of phi and alpha with x and y shoulld be part of this set.
I think those are important as they define the encircling in parameter space.

3) Fig. 2: Is the first point at wt=0 measured. I guess its 1 just by normalization and does not carry any "experimental" information, maybe skip it?
If the experimental time resolution would be better will there still be a step or will it be smeared out?
Can you give the values of the horizontal dashed lines (fitting) of the four and discuss their differences?

How are the experimental times are defined. I would have thought that the time taken is at central times of delta t, this there would be none at wt=0, wt=2pi and probably at wt=pi neither?
See also last paragraph before data availability statement (page 6).
Is (w\delta t) so small?
Why don't you plot all data points measured or are those 20 points the only ones measured? In this case what is N=5x10^4 means?
I could not find omega and \delta t (only 266.7 ns laser pulse and 1550 nm).

4) The precision of the experimental initial state is announced to be 0.05% meaning that the deviations observed would not be visible. Thus the explanation for this deviation cannot be the initial state?

Remarks for the Supplementary:

Why is Fig. S2 using a different order than in the other figures?
The theoretical value of lower limit of the fidelity could be stated. Is it the Peterman factor.
Is it the same as in Fig. S7 (maybe use the same y-axis label values)
The agreement of the experiment with the theory is nto very good. In (a) red points I would even think a linear function would match better. Looks like the fidelity is more sensitive than the relative eigenstate populations.
Can you explain this?

I'm missing details on the time steps etc. to obtain Fig. S6 (and S8).
Why is the value -1 is approached closest after the jump and not at 100

Will it be possible to plot the polarization components of the measured states directly in the supplementary, without projection, thus the measured data is presented?

Reviewer #3

(Remarks to the Author)
The manuscript "Observation of Restored Adiabatic State Transfer in
Time-Modulated Non-Hermitian Systems" reports the first experimental demonstration of restored adiabatic state transfer in a non-Hermitian system. Traditionally, encircling an exceptional point leads to chiral mode switching caused by complex energy spectra. In this work, authors utilize a specific trajectory in parameter space that constrains the evolution operator to a purely real spectrum, thereby suppressing non-adiabatic transitions and enabling symmetric, adiabatic state transfer regardless of the path direction. Using a photonic polarization platform, they further demonstrate controllable switching between symmetric and chiral state
transfer within the same experimental architecture. The work is experimental validation of recent theoretical predictions (Phys. Rev. Lett. 133.11 (2024): 113802).

Comments: The results are of broad interest to the non-Hermitian, photonics, and quantum control communities. I find the manuscript well within the scope of Nature Communications.

I have following questions/comments for authors:

1. Parameters x and y are introduced mathematically in Eq.(2) to define the trajectories. Since these parameters are central to the claim of restored adiabaticity, a clearer explanation of mapping (x, y) -> (epsilon, k, kappa, both epsilons) and some intuitive explanation beside this abstract parametrization would significantly improve overall clarity.

2. Given the experimental nature of this work, the authors should provide a more explicit description of how the set of Hamiltonian parameters {gain/loss (epsilon), dissipative coupling (kappa), coherent coupling (k) and detuning} are controlled in the experimental setup in the main text.

3. The adiabatic restoration relies on a specifically designed trajectory in parameter space. How sensitive is the adiabaticity to small deviations from this trajectory? Does a slight departure immediately re-introduce chiral behavior?

4. As I understand, the adiabaticity criteria used in this work is the absence of non-adiabatic transitions. Are there any constraints on how fast the loop can be traversed? To be specific, I am referring to \omega in Eq.(3).

5. The data in Fig. 2 of the main text appear noticeably less noisy than the corresponding results shown in Fig. S2. Could the authors clarify the origin of this difference?

6. Since the present implementation relies on classical coherent light and photodetection, a brief discussion on the prospects and potential challenges of extending this approach to a quantum regime, where quantum noise may become relevant, would be useful.

7. The manuscript states that the demonstrated symmetric adiabatic state transfer may enable applications in topological quantum computation and related quantum information platforms. It is further claimed in the Supplementary Material that this protocol has potential applications in quantum communication, quantum computing, and quantum networks. These claims

are rather general. It would strengthen the manuscript if the authors could provide concrete examples of specific tasks which would benefit from demonstrated protocol, or how this mechanism could be incorporated into realistic photonic or quantum-information architectures.

Minor comments:
8. The choice of notation in Eq. (1) could be improved. Several symbols (e.g., k and κ, ϵ and ε) are visually similar and may lead to confusion and mix-up. Moreover, 'k' is repeated in the eigenstate label \ket{psi_k} elsewhere in the text.

9. Do the colors in Fig. S1 represent a physical quantity, or are they included purely for aesthetic purposes?

Reviewer #4

(Remarks to the Author)
I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

Reviewer #5

(Remarks to the Author)
In this work, the authors simulated adiabatic state transfer by encircling an exceptional point (EP) in a non-Hermitian system using the two polarizations of a laser. Their approach entails simulating a sequence of individual non-unitary adiabatic evolution operators, $U(t_n)$, for discrete time slices t_n. By decomposing $U(t_n)$ as $R_1 L R_2$, the unitary operations R_1 and R_2 can be realized using a standard wave plate setup, while the diagonal matrix L can be implemented using components such as an erbium-doped fiber amplifier and optical delay lines. However, this approach does not seem to actually realize a full adiabatic state transfer process, as it simulates only a sequence of discrete, individual non-unitary evolution operators, $U(t_n)$.

That said, the ability to experimentally access an EP encircling trajectory is still of high potential scientific value. Given the substantial body of work on state transfer by encircling EPs in non-Hermitian systems [e.g., Phys. Rev. Lett. 86, 787 (2001); Nature 537, 76 (2016)], the authors should clearly articulate the significance and novelty of this work i.e. what distinguishes their approach from previous implementations or whether they have overcome specific technical challenges.

Below are some additional comments:

1. The manuscript repeatedly states that the evolution operator acquires a purely real spectrum along the designed EP-encircling trajectories. It would be good to clearly explain the necessity of this. Since the same photon does not actually undergo repeated U(t) slice evolutions, some loss is ok too?

2. he use of 21 discrete time points is sufficient for the purpose of the main figures, i.e., to identify the direction-independent final state after a full encircling cycle, since the reported P(t) dynamics is largely piecewise constant with a clear switching outcome. However, the more critical issue for the strong claim of "restored adiabaticity" is not the temporal sampling density, but the lack of a systematic validation with respect to control parameters and numerical/experimental discretization. In particular, since the time-ordered evolution is approximated as a product over N segments (with N = 5 × 10^4 chosen in the experiment), the authors should demonstrate that the reported state-transfer outcome converges with respect to N, and that the adiabaticity metrics improve as the protocol is slowed (e.g., by scanning T or omega), for both encircling directions. Robustness checks against loop parameters would further strengthen the generality of the "restoration" claim. Since this may require substantial additional experimental efforts, a numerical demonstration would also suffice.

5. The manuscript repeatedly emphasizes a "genuine programmable symmetric–chiral (symmetric–asymmetric) two-mode switch," "long thought unattainable" and "universal." However, the demonstrated "switching" is realized by implementing two distinct effective Hamiltonians, H in Eq2 and H0 in Eq4, within the same optical platform, with the two models coinciding only at t = 0 by construction for comparability. Please clarify in what precise sense the switch is "programmable" and "universal": is there a single continuous control knob within a single Hamiltonian family that tunes the system from symmetric/adiabatic to chiral/non-adiabatic transfer, or is the programmability mainly due to the platform's ability to emulate different models? If the former is intended, it would be important to provide evidence of a continuous crossover (e.g., by gradually introducing the imaginary-part spectrum or controlled deviations from the constraint manifold) rather than presenting two separate model realizations.

Reviewer #6

(Remarks to the Author)
I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

Version 1:

Reviewer comments:

Reviewer #1

(Remarks to the Author)
My detailed comments on the revised manuscript and the authors' responses are provided in the attached PDF.

Reviewer #2

(Remarks to the Author)
Second report for 'Observation of Restored Adiabatic State Transfer in Time-Modulated Non-Hermitian Systems' by Xiaowei Wang, Ievgen I. Arkhipov, Quan Lin, Huixia Gao, Dengke Qu, Lei Xiao, Franco Nori, and Peng Xue (NCOMMS-25-90175).

The authors have well replied to my remarks (and the ones from the other referees) and I think deserves to be published in Nature Communications but would like that it is stressed that it is not an "in-situ" encircling.

Still I'm a bit confused about the way the encircling is performed "experimentally".
I think it should be clearly stated that the experiment is not an "in-situ" encircling but is done via numerically multiplying experimental results. The authors detail how the U operator is adjusted using numerical simulations but the statement that at the end 21(?) measurements are done giving \psi_i^{exp} measured states (with averaging?) and the final state is generated by multiplying this states. Is this correct?
Concerning the error bars are they obtained using the information of several experimental runs (i.e. standard deviation of N_exp samples, then give numbers) or obtained by estimating errors by precision of the elements?

The statement in the conclusion "To the best of our knowledge, we report the first experimental demonstration that the adiabatic state evolution can be fully restored when dynamically encircling an EP, regardless of the winding direction." should be modified (don't know whether first experimental is valid in nature communications anyway) as this implies that the experiment is performing the dynamically encircling "in-situ" which it does not!
Thus a statement would be more like:
we report experimental evidence that ... by measuring the individual linearized evolution and generate the dynamical encircling in the computer by multiplying the experimental results ...

Reviewer #3

(Remarks to the Author)
The authors have addressed the referee concerns. At this point, I recommend publication in Nature Communications.

Reviewer #4

(Remarks to the Author)
I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

Reviewer #5

(Remarks to the Author)
The revised manuscript has been significantly improved and the main concerns raised previously have been adequately addressed. The additional discussions in the revised manuscript and Supplementary Material make the physical interpretation much clearer. I therefore support publication of this work.

I only have one minor comment. Since the term “adiabatic” may still invite the conventional Berry-Kato interpretation, the authors may consider adding one brief reminder that here it is used in the operational sense of the absence of discontinuous

non-adiabatic transitions.

Reviewer #6

(Remarks to the Author)
I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

*** REPORT OF THE REFEREE 1 ***

REFEREE'S COMMENT I

The manuscript reports an experimental realization of adiabatic and symmetric state transfer in a time-modulated non-Hermitian two-level system by dynamically encircling an exceptional point. The authors show that non-adiabatic transitions can be suppressed when the evolution operator exhibits a purely real spectrum, leading to direction-independent eigenstate exchange. The topic is timely and relevant within the context of non-Hermitian physics, and the experimental implementation appears technically sophisticated, extending previous studies on chiral mode switching associated with complex spectra.

OUR REPLY

We thank the referee for this positive summary of our work and for dedicating time to the thorough review of our manuscript.

REFEREE'S COMMENT II

At the same time, the current presentation may leave room for ambiguity in the interpretation of the reported state exchange.

Overall, the manuscript addresses an appealing physical idea and presents technically impressive experimental results. However, in its current form, the distinction between representational features and intrinsic spectral properties is not always evident, particularly in connection with the use of the term "spectrum," the appearance of branch-cut–like structures, and the interpretation of adiabaticity. Making these distinctions more transparent would strengthen the conceptual clarity of the work and reduce the risk of misinterpretation.

OUR REPLY

We appreciate the referee's positive assessment and helpful suggestions. Our point-by-point responses and the associated manuscript revisions are detailed below.

REFEREE'S COMMENT 1

In the manuscript, the Hamiltonian depends on a two-dimensional set of control parameters (x, y), and the eigenenergies discussed correspond to instantaneous values evaluated along this parameter dependence. The use of the term "spectrum" in this context therefore refers to these parameter-dependent eigenvalues, but its relation to the conventional notion of a spectral structure is not explicitly specified.

OUR REPLY

We thank the referee for pointing out this lack of clarity. In the revised manuscript, we have explicitly clarified the distinction between the full spectrum of the non-Hermitian Hamiltonian (NHH) H_0 in Eq. (1), whose eigenvalues are given in Eq. (2), and the spectrum of the parameterized NHH H in Eq. (4), whose eigenvalues are defined in Eq. (5).

We stress that although the parameterized Hamiltonian H depends on a reduced set of control parameters (x, y), the corresponding eigenvalues retain the status of a conventional spectrum. Indeed, through the hyperbolic embedding introduced in Eq. (3), each point (x, y) is mapped one-to-one onto a unique point in the full four-dimensional parameter space $(\epsilon, k, \varepsilon, \kappa)$, and therefore specifies a unique Hamiltonian (we note that in the revised manuscript, for the sake of clarity, we have renamed the system parameters, namely $k \to u$ and $\varepsilon \to v$). Consequently, the eigenvalues $E_{1,2}(\alpha)$ in Eq. (5) are uniquely defined for each parameter configuration and do not arise from representational choices, branch selections, or projections. They thus represent a well-defined real-valued subset of the full complex spectrum $E_{\pm}$.

To reflect this clarification, we have revised the text in the section *Theory* [page 2, the 2nd column, after Eq. (5)]:

Specifically, the system parameters in Eq. (3) define a two-dimensional (2D) hyperboloid embedded in the 4D parameter space (ϵ, u, v, κ) satisfying $\epsilon^2 + u^2 - v^2 - \kappa^2 = \alpha^2$ (see also the Supplementary Material for an intuitive geometric visualization of this manifold). Importantly, by restricting the system parameters to such a 2D manifold within the full 4D parameter space, one can effectively reduce the NHH H_0 in Eq. (1), which generally exhibits complex eigenvalues, into a parameter regime where the spectrum becomes purely real. Owing to the hyperbolic embedding defined by Eq. (3), each point (x, y) uniquely determines a single point in (ϵ, u, v, κ) in the full parameter space, and hence a unique Hamiltonian. Therefore, the spectrum of the NHH H is well defined. Physically, this restriction enforces a balance between gain and loss that removes the imaginary part of the spectrum.

REFEREE'S COMMENT 2

This issue is particularly evident in Fig. 1(a), where a line-like feature appears that is visually reminiscent of a branch cut in the conventional sense associated with complex multivalued functions. When viewed strictly within a two-dimensional parameter-space picture, such a structure could be taken to suggest an extended real-energy degeneracy, which would prevent adiabatic following. By contrast, branch cuts associated with complex eigenenergies do not necessarily preclude adiabatic evolution. At the same time, this feature may arise from how higher-dimensional parameter dependence is represented in two dimensions, from conventions used to label eigenstates, or from how continuity is enforced in the chosen representation. Clarifying the physical meaning of this branch-cut–like structure, whether it reflects an intrinsic spectral property or instead results from representational choices, is therefore important for how the figure is understood.

OUR REPLY

We thank the referee for this important comment. The branch-cut-like feature visible in Fig. 1(a) can indeed be ambiguous if interpreted in direct analogy with branch cuts associated with complex spectra. We therefore agree that clarifying its physical meaning is essential for a correct understanding of the figure and of the adiabaticity discussed in this work.

The line-like structure appearing in Fig. 1(a) does not arise from representational choices, eigenstate labeling conventions, or from projecting a higher-dimensional parameter dependence onto two dimensions. Rather, it reflects an intrinsic spectral property of the reparameterized pseudo-Hermitian Hamiltonian H: the branch cut coincides with a diabolic line, i.e., an extended real-energy degeneracy where $E_{1,2} = 0$ in Eq. (5). In this sense, the branch cut should be understood as a genuine degeneracy in the real-valued spectrum, rather than as a conventional branch cut associated with complex Riemann-sheet topology.

This observation naturally calls for clarification of the notion of adiabaticity, as noted by the referee. Because the evolution involves a diabolic degeneracy, adiabaticity in the strict Berry–Kato sense, which is valid away from a spectral gap closing, is not applicable throughout the trajectory. The protocol considered here does not rely on this form of adiabaticity, and this distinction is now made explicit in the revised manuscript.

In our work, by the term "restored adiabaticity" we mean the suppression of discontinuous non-adiabatic transitions (NATs) that typically arise in dynamical EP-encircling protocols with complex spectra and lead to chiral, direction-dependent state transfer. In the present protocol, such NATs are absent and the state evolution remains continuous (as supported by Fig. S3 in the Supplementary Material). This behavior is enabled by the fact that, along the encircling trajectory, the spectrum of H remains purely real, which suppresses mode-selective amplification and prevents the dynamical instabilities responsible for NATs.

To avoid any possible confusion, we have revised the manuscript by adding several paragraphs immediately before the *Results* section, where we explicitly clarify below.

The interpretation of the branch-cut in Fig. 1(a) as a diabolic line rather than a representational artifact (page 2, the last paragraph):

> Interestingly, the branch cut ($x = 0, -1 < y < 1$) coincides with a diabolic line defined by the condition $E_{1,2} = 0$ in Eq. (5). As a consequence, the EP-encircling

> protocol with restored *adiabaticity* considered here, illustrated in Fig. 1(a) and discussed in detail below, necessarily involves traversing this diabolic degeneracy.

The precise dynamical meaning of adiabaticity employed in this work, clearly distinguishing it from adiabaticity in the Berry–Kato sense (page 3, the 1st paragraph):

> This observation motivates a clarification of the notion of adiabaticity used in the present protocol. In particular, adiabaticity in the Berry–Kato sense, which is based on the following instantaneous eigenstate in the presence of a finite spectral gap, is not directly applicable here. This situation differs from conventional EP-encircling schemes in systems with complex spectra, where branch cuts do not generally coincide with extended real-energy degeneracies and the assumptions underlying the standard adiabatic theorem may still be fulfilled. Here, 'adiabatic' therefore refers exclusively to a dynamical regime in which sufficiently slow parameter modulation yields continuous state evolution, with adiabaticity identified operationally through the absence of NATs. This dynamical behavior is enabled by the absence of an imaginary component in the spectrum of the NHH H along the encircling trajectory, which suppresses mode-selective amplification and thus prevents *discontinuous* NATs that typically occur away from the branch cuts [27]. In what follows, unless explicitly stated otherwise, we use the term adiabaticity exclusively in this operational sense.

We note that, in this context, the EP-encircling protocol discussed and experimentally implemented here closely parallels the adiabatic rapid passage (ARP) protocol familiar from Hermitian systems. In ARP, robust state transfer is achieved by slowly sweeping system parameters so that the state evolves adiabatically away from a diabolic degeneracy and can be continuously transferred across the degeneracy itself.

From this perspective, the protocol realized here can be understood as a genuine non-Hermitian extension of ARP. While the conceptual analogy between dynamical EP encirclement and ARP has been previously discussed and experimentally explored in a recent work (newly cited Ref. [45] in the revised manuscript), those realizations involve complex spectra and are inevitably accompanied by NATs, preventing a fully continuous dynamical evolution and thus obscuring a direct identification with ARP.

By contrast, in the present work, the EP encirclement is performed within a parameter regime where the spectrum remains purely real, thus implying suppressed NATs, and where the branch cut corresponds to a true diabolic line through which the driven state is continuously transferred.

To further stress this point, we have added a new paragraph (page 4, the 2nd paragraph):

> In this respect, the symmetric state-switching protocol based on dynamical EP encirclement in such systems with a purely real spectrum echoes the adiabatic rapid passage protocol familiar from Hermitian systems, where robust state transfer can be similarly achieved through slow parameter variation by traversing diabolic degeneracies during the state evolution [44]. Similar parallels between these Hermitian and non-Hermitian state-flipping protocols have already been discussed and experimentally explored in Ref. [45]. Crucially, however, compared to Ref. [45], the protocol experimentally implemented here operates in a parameter regime where the diabolic line emerges as the branch cut terminating at EPs [see Fig. 1(a)], and where the continuous state evolution remains entirely free of NATs.

REFEREE'S COMMENT 3

> A closely related point arises in the interpretation of the dynamical results shown in Fig. 2. There, an abrupt change in the identified eigenstate occurs at a location that coincides with the branch-cut–like structure in the two-dimensional representation. Such behavior may be understood as resulting from how instantaneous eigenstates are defined or tracked within the chosen framework, rather than from a genuine spectral singularity encountered during the evolution. If this distinction is not clearly stated, the observed transition may appear to be directly associated with a spectral gap closing, even though the underlying mechanism may be different.

OUR REPLY

We thank the referee for the opportunity to further clarify this point. We confirm that the seemingly abrupt change of the eigenstate observed in Fig. 2 occurs precisely at the location of the diabolic line shown in Fig. 1(a). Importantly, this behavior does not indicate a dynamical singularity; rather, it reflects the continuous evolution of the system across this diabolic degeneracy, at which the instantaneous eigenstates *exchange their ordering*. In the convention adopted throughout the manuscript (as stated below Eq. (5) in the revised manuscript), the eigenvalues are labeled such that $E_1 \geq E_2$, and it is this labeling choice that leads to the apparent abrupt switch at the diabolic line.

To support this claim, in the original version of the Supplementary Material, we have already plotted the state evolution projected on the initial states, such that its continuous dynamics can be seen vividly. To further stress this point, we have modified the following text in the revised manuscript (page 5, the 2nd column, the 2nd paragraph):

> We note that the apparent abrupt change of the projection of the evolving state on the instantaneous eigenstates at $t = T/2$ in Fig. 2, corresponding to the crossing of the diabolic line, arises from the interchange of the instantaneous eigenstates at this degeneracy, in accordance with the adopted labeling convention $E_1 \geq E_2$. Importantly, the state evolution remains fully continuous across this degeneracy, as further illustrated by complementary fidelity plots between the evolving and initial states provided in the Supplementary Material.

REFEREE'S COMMENT 4

> The Supplementary Material introduces a description in a four-dimensional parameter space, in which features that appear singular in the two-dimensional projection correspond to smooth structures with a finite complex spectral gap, except at isolated exceptional points. This higher-dimensional perspective provides useful context for interpreting the two-dimensional figures and for understanding how adiabatic evolution is restored in the present system. However, the role of this description in relation to the main text is not entirely clear.

OUR REPLY

We thank the referee for this useful comment. In the revised manuscript, we have added further details on the parameterized 2D hyperboloid manifold embedded within the system's 4D parameter space directly in the main text, with an explicit reference to Fig. S1 in the Supplementary Material. This now makes the connection between the discussion in the main text and Fig. S2 clear and transparent.

Namely, the text in the revised manuscript [right below Eq. (5)] reads:

> Specifically, the system parameters in Eq. (3) define a two-dimensional (2D) hyperboloid embedded in the 4D parameter space (ϵ, u, v, κ) satisfying $\epsilon^2+u^2-v^2-\kappa^2 = \alpha^2$ (see also Supplementary Material for an intuitive geometric visualization of this manifold).

REFEREE'S COMMENT 5

> These points are particularly relevant given that the experimental protocol implements time evolution in a discretized, stepwise manner, where the interpretation of adiabaticity is tied to the instantaneous spectral structure along the driven path. When features appearing in reduced or projected representations may reflect parametrization or visualization choices rather than intrinsic spectral properties, this distinction becomes important for interpreting the observed state exchange. As an optional point, not essential to the main claims, examining how the behavior varies with the driving or modulation rate could help further place the experiment within an adiabatic context and may also provide indirect quantitative information about the effective minimum spectral gap along the trajectory, even when this gap is not directly visible in the two-dimensional representation.

OUR REPLY

We appreciate this very useful suggestion from the referee. In the revised Supplementary Material, we have added new numerically simulated fidelity plots in Fig. S12, which

complement the experimental data shown in Fig. S3. These simulations present the fidelity of the evolving state for different values of the winding period T. The newly added plots demonstrate that, in the adiabatic (slow-driving) limit $T \to \infty$, the state evolution remains continuous and the state-switching protocol is robust. By contrast, for sufficiently short winding periods, diabatic effects emerge, leading to deviations from the ideal evolution along the eigenenergy manifolds and a breakdown of the state-switching protocol.

***** REPORT OF THE REFEREE #2 *****

REFEREE'S COMMENT I

The authors present in their manuscript an optical experimental realization to encircle an EP but keeping an adiabatic state transfer which is based on a theoretical work [33]. The basic idea is to encircle the EP on a purely real Riemanian sheet. They find a good agreement between their experimental results and the theoretical predictions and also show that in case of using a complex sheet the non-adiabatic behaviour is found.

OUR REPLY

We thank the referee for the favorable summary of our results and for the time dedicated to reviewing our manuscript.

REFEREE'S COMMENT II

I find the paper interesting, well written, and think deserves to be published in Nature Communications if two issues can be explained.

OUR REPLY

We thank the referee for the overall positive assessment of our work. Below, we address the comments.

REFEREE'S COMMENT 1

1) Is the experiment really encircling the exceptional point dynamically (I can see this in the setup of the appendix, p4) but from the experimental side I have the impression that its just two lines (one polarization going through VOA, the other through DL) and they interfere via the PBC. Thus at the moment for me the experiment is mimicking the results of an EP-encircling and not really

performing the encircling! Maybe to address this question a reduced scheme in fig 1 will be helpful just detailing the principle leaving out BPF, EDFA, VPC etc.. This will be helpful for readers not coming from optics anyway. Also it might be helpful what are the experimental parameters that are varied (ratios etc). How many experiments have been performed for a single point? Why are the error bars only from the polarimeter, is it experimental or the precision of the device? To be very precise how are the $N = 5 \times 10^4$, $U(t)$ are applied for the $wt = 2\pi$ measurement. Where is the feeding in of the U_n outgoing signal to the $U_{(n+1)}$ ingoing signal. If I understand correctly the VOA is applying a single U_n?

OUR REPLY

We thank the referee for the important questions, which allow us to clarify the experimental implementation and the sense in which the continuous EP-encircling dynamics is realized in our work.

In our study, the experiment effectively simulates the continuous evolution generated by a time-dependent NHH $H(t)$ through a family of static non-unitary operators $U(t)$, rather than performing a continuous physical recirculation of light. For a given initial state, each target time t_q corresponds to a single operator $U(t_q)$, which is physically implemented once in the optical setup (using the modules including VPC, VOA, DL, EDFA, etc.), i.e., the light passes once through the optical setup, where the output and input signals are related via the relation $|\Psi(t)\rangle = U(t)|\Psi_0\rangle$. The multiple integration steps $N = 5 \times 10^4$ used to construct $U(t_q)$ are purely numerical: they approximate the time-ordered exponential $\mathcal{T} \exp[-i \int_0^{t_q} H(t), dt]$ and do not correspond to discrete experimental operations. In particular, there is no sequential feeding of U_n into U_{n+1}.

This experimental approach used allows us to faithfully emulate the continuous dynamics of the system for any chosen t_q without relying on incremental or stroboscopic sweeps of Hamiltonian parameters, as done in previous experiments. By repeating the measurement for all 21 target times $(t_0, \ldots, t_{20})$, we reconstruct the evolution along the EP-encircling trajectory.

To further highlight the principle of our experimental setup and dispel any confusion,

we have added new paragraphs in the revised manuscript (page 4, the 2nd column, the 2nd paragraph):

For a given initial polarization state $|\Psi_0\rangle$, the system evolution is described by

$$|\Psi(t)\rangle = U(t)|\Psi_0\rangle,$$

where $|\Psi(t)\rangle = [a(t), b(t)]^T$ represents the polarization state at evolution time t, with $a(t)$ and $b(t)$ denoting the horizontal and vertical field components, respectively.

In the experiment, we simulate the continuous system dynamics, generated by a time-dependent NHH $H(t)$, by constructing a family of "static" non-unitary evolution operators $U(t)$, each of which corresponds to the time evolution from $t = 0$ to a chosen target time t. Specifically, in the experiment, we select 21 representative target times $(t_0, \ldots, t_{20})$. Each target time t_q corresponds to an independently constructed operator $U(t_q)$. For example, for $\omega t_q = \pi$, the operator $U(t_q)$ represents the evolution from $t = 0$ to $t_q = \pi/\omega$. This operator is then applied to the initial state in a single experimental run (as shown in Fig. 1). As such, hereafter, unless stated otherwise, the terms "time evolution" and "system dynamics" refer exactly to this simulated evolution.

To obtain each $U(t_q)$, we first numerically integrate the Schrödinger equation from $t = 0$ to $t = t_q$. This integration is performed using $N = 5 \times 10^4$ time steps, which ensures the convergence of the time-ordered exponential (see also Supplementary Material). Formally, the evolution operator is approximated as

$$U(t_q) \approx \prod_{n=1}^{N} \exp[-iH(t_n)\,\delta t]\,,$$

where $\delta t = t_q/N$ and $t_n = (n - \frac{1}{2})\delta t$ denotes the midpoint of each integration interval. Each such numerically constructed operator $U(t_q)$ is then physically implemented by appropriately configuring the optical elements, thereby directly mapping the integrated evolution onto the output state. Repeating this procedure for different target times t_q allows us to reconstruct the continuous dynamics of the system without relying on incremental or stroboscopic parameter modulation.

The experiment thus physically implements this specific sequence of operators using variable polarization controllers and a split-path gain/loss module using the two-mode rail optical platform (Fig. 1(b)). The experimental parameters varied for each data point are the rotation angles in the two VPCs (to implement unitary operators R_1 and R_2), and the gain/loss ratios related to the attenuation of the VOA and the gain of the EDFA together with DL(to implement the diagonal matrix L). These are derived explicitly from the singular value decomposition of $U(t)$, which is calculated using the parameters of the system's Hamiltonian. This approach allows us to reconstruct the state trajectory with high fidelity, avoiding the cumulative noise of long-loop light evolution. In the experiment, for each experimental data point, we perform a total of $5 \times 512 = 2560$ measurements. Here, the factor of 5 arises because we repeatedly extract five sets of complete polarization information from the polarization state reconstruction. For each set, the value of 512 corresponds to the default number of measurement points that the polarimeter employs during the rotation of a quarter waveplate for polarization analysis, which is sufficient to fully determine the state of polarization. We clarify that the error bars do not originate solely from the polarimeter itself. Instead, they represent overall experimental statistical errors, which include contributions from instrument instability, environmental fluctuations, and disturbances introduced by the optical fibers. Thus, the error bars reflect comprehensive experimental uncertainties, rather than only the precision of the measurement device. We have already clarified this in the caption of Fig. 2.

Moreover, we are aware of the distinction between the efficient simulation and full continuous dynamics. In fact, as detailed in the Supplementary Material (the section titled *"A Proposed Experimental Scheme for the Dynamical Observation of the Adiabatic State Transfer"*), we have expanded our two-line platform and explicitly proposed the fiber-loop scheme (Fig. S6) where the pulse physically circulates, and each round trip corresponds to a stroboscopic simulation $e^{-iH(t_n)\delta_t}$. This procedure can be viewed as a genuine quantum simulation of the dynamics, rather than a classical simulation based on measurement followed by state re-preparation. We also perform numerical simulations for this dynamical scheme (Fig. S7 and Fig. S8) which confirm our findings. However, as noted in the text, the continuous implementation requires ultra-fast polarization modulation devices and precise temporal synchronization, which introduces significant experimental complexity and noise. For this reason, we expect this type of implementation to become more practical in future

experiments, when optical components and control technologies are further advanced.

Therefore, for this proof-of-principle demonstration of restored adiabaticity, we choose the simulation approach to maximize control precision and state fidelity. This method allows us to reconstruct the exact state that the system would occupy if it had evolved dynamically up to time t. This approach offers superior control and avoids the decoherence and dispersion accumulation typical of long physical fiber loops, allowing for a precise verification of the topological protection mechanism. This constitutes the first genuine programmable symmetric-chiral state switch, an ability long considered hard to implement in two-level systems.

To further clarify this point, we have added the corresponding clarification in the *Results* section (page 4, the 2nd column, the last paragraph):

> In the experiment, we implement this sequence of operators using variable polarization controllers and a split-path gain–loss module within a two-mode optical platform ....

And in the text (page 5, the 1st column, the 1st paragraph):

> ... The experimental parameters varied for each data point are the rotation angles in the two variable polarization controllers to implement two unitary operators, and the gain/loss ratios related to the attenuation of the variable optical attenuator and the gain of the erbium-doped fiber amplifier to implement the operator L. These are derived explicitly from the singular value decomposition of $U(t)$, which is calculated using the parameters of the system's Hamiltonian. This allows us to reconstruct the state trajectory with high fidelity, avoiding the cumulative noise of long-loop light evolution associated with a continuous implementation.

And also in the *Methods* section of the revised manuscript (page 8, the 2nd column, the last paragraph):

> ... Here, N refers to the number of steps in numerical integration (the total number of segments as mentioned above) used in the theoretical calculation of the target operator $U(t)$. We use a large N to ensure the theoretical matrix is computed with high precision, rather than feed the signal back 5×10^4 times

in the experiment. ... The deviations are dominated by experimental imperfections in implementing the non-unitary evolution operator and measurement uncertainties, which include contributions from instrument instability, environmental fluctuations, and disturbances introduced by the optical fibers.

Following the referee's suggestion, we have also added a simplified schematic to Fig. S2 in the revised Supplementary Material to clarify the single-pass principle of our efficient dynamics simulation.

REFEREE'S COMMENT 2

2) I'm missing a detailed discussion on the effects of the linearization and discretization of the time steps. This implies that the trajectory is not perfectly sitting on the real Riemannsheets but slightly go into the complex parts. The theoretical discussion to perform this adiabatic state transfer around the EP is based on the fact that one stays "always" on the real sheets. Thus it might be more related on how fast the adiabatic process is. There have been a paper which displays this (also experimentally): PHYSICAL REVIEW A 102, 040201(R) (2020) Encircling exceptional points as a non-Hermitian extension of rapid adiabatic passage DOI: 10.1103/PhysRevA.102.040201 Could you please detail the difference of this work and the rapid adiabatic passage scheme. In any case the paper should be cited and discussed in the introduction as it displays the same result as the authors are aiming for in this manuscript.

The replies to this two issues are crucial to see whether its novelty is sufficient to be published in nature communications.

OUR REPLY

We thank the referee for raising this important point. As highlighted in our response to the previous comment, in the experiment we choose a small discrete time interval and a slow evolution speed to ensure accurate convergence and a faithful approximation of the underlying continuous-time dynamics. This discretization is sufficient to keep the system

dynamics confined to the real-valued eigensheets, thus avoiding dynamical instabilities associated with non-adiabatic transitions (NATs) that could arise from the emergence of complex eigenvalues.

This behavior is further supported by the experimental results shown in Fig. 2 and Fig. S3, as well as by the accompanying numerical simulations, which consistently demonstrate a genuinely continuous and symmetric state-switching protocol enabled by the effective suppression of NATs, following real-valued eigenenergies.

We also thank the referee for drawing our attention to this relevant reference [Physical Review A 102, 040201(R) (2020)], which we have now cited as Ref. [33] in the revised manuscript. In addition to establishing parallels between the adiabatic rapid passage (ARP) protocol in Hermitian systems and EP-encircling protocols in non-Hermitian systems, Ref. [33] identifies a minimal critical amount of loss required to induce non-adiabatic transitions (NATs), determined by the *specific* loop trajectory and the encircling period, according to Eq. (4) therein. Moreover, as the authors stressed in Ref. [33], the loss itself must be small $\gamma \ll 1$ to validate the latter equation.

Crucially, in the protocol experimentally implemented in our work, no such restrictions are imposed on the winding period T and the loop shape in the (x, y)-space, provided that the time evolution is sufficiently slow to avoid fast diabatic effects. Moreover, there are no intrinsic limitations on the system dissipation rate (gain–loss). Indeed, as follows from Eqs. (4) and (5) of the revised manuscript, the energy α can be scaled to arbitrarily large values, corresponding to strong dissipation (diagonal imaginary values in the NHH H), while the spectrum of the Hamiltonian nevertheless remains entirely real.

In this sense, and in line with the discussion in Ref. [33], the symmetric state-switching protocol realized in our work can indeed be viewed as a natural extension of the ARP protocol: the branch cut connecting real-valued Riemann sheets plays the role of a diabolic line, and encircling the EP while crossing this line enables symmetric eigenstate switching without observable NATs in the system dynamics.

Importantly, unlike conventional ARP implementations, our protocol does not require alternating the system parameters on and off when going around and sweeping across the diabolic degeneracy. Instead, it relies on a smooth and continuous modulation of system parameters, thus avoiding issues related to pulse timing and synchronization which are, in general, inherent to ARP schemes. To further clarify this point, we have added the

corresponding discussion to the revised manuscript (page 4, the 2nd paragraph):

> In this respect, the symmetric state-switching protocol based on dynamical EP encirclement in such systems with a purely real spectrum echoes the adiabatic rapid passage (ARP) protocol familiar from Hermitian systems, where robust state transfer can be similarly achieved through slow parameter variation by traversing diabolic degeneracies during the state evolution [45]. Similar parallels between these Hermitian and non- Hermitian state-flipping protocols have already been discussed and experimentally explored in Ref. [33]. Crucially, however, compared to Ref. [33], the protocol experimentally implemented here operates in a parameter regime where the diabolic line emerges as the branch cut terminating at EPs [see Fig. 1(a)], and where the continuous state evolution remains entirely free of NATs.

And newly added paragraph (page 5, the 2nd column, the 3rd paragraph):

> Importantly, the symmetric state-switching protocol demonstrated here is not restricted to specific winding periods T or particular EP-encircling trajectory shapes in the (x, y)-space [44]. Instead, it remains effective for a broad range of system parameters in Eq. (6), provided that the system evolution is sufficiently slow to suppress fast diabatic effects (see also Supplementary Material). This behavior differs from the scenario considered in Ref. [33], where maintaining the adiabatic character of the EP-encircling protocol requires additional constraints on the system parameters.

REFEREE'S COMMENT 3

> Detailed remarks:
>
> 1) The statement "ultra-high-precision sensing [12–14]," should be made with care as nowadays it is more or less agreed that due to noise enhancement this ultra high sensing is not possible (thought sensors based on EPs might still be useful/partially outperforming other)?

OUR REPLY

We thank the referee for this important remark. We agree that claims of ultra-high-precision sensing in the context of exceptional points should be treated with caution, particularly in light of noise amplification effects that can limit the achievable sensitivity. In the revised manuscript, we have therefore modified this wording and replaced the phrase "ultra-high-precision sensing [12–14]" with the expression "enhanced sensing [12–14]."

REFEREE'S COMMENT 4

2) I think the equations for epsilon, ..., kappa should be given an equations number and the relations of phi and alpha with x and y should be part of this set. I think those are important as they define the encircling in parameter space.

OUR REPLY

We thank the referee for this useful suggestion. In the revised manuscript, we have introduced the explicit mapping from the four-dimensional system parameter space to the two-dimensional hyperboloid surface as a separate Eq. (3). This equation is now also referenced in the main text to improve clarity of the introduced mapping. Namely, below this new Eq. (3) we have added the following text (for the sake of clarity, we have renamed the system parameters in the revised manuscript, namely $k \to u$ and $\varepsilon \to v$):

Specifically, the system parameters in Eq. (3) define a two-dimensional (2D) hyperboloid embedded in the 4D parameter space (ϵ, u, v, κ) satisfying $\epsilon^2 + u^2 - v^2 - \kappa^2 = \alpha^2$ (see also Supplementary Material for an intuitive geometric visualization of this manifold). Importantly, by restricting the system parameters to such a 2D manifold within the full 4D parameter space, one can effectively reduce the NHH in Eq. (1), which generally exhibits complex eigenvalues, into a parameter regime where the spectrum becomes purely real. Owing to the hyperbolic embedding defined by Eq. (3), each point (x, y) uniquely determines a single point in (ϵ, u, v, κ) in the full parameter space, and hence a unique Hamil-

tonian. Therefore, the spectrum of the NHH H is well defined. Physically, this restriction enforces a balance between gain and loss that removes the imaginary part of the spectrum.

REFEREE'S COMMENT 5

3) Fig. 2: Is the first point at $\omega t = 0$ measured. I guess its 1 just by normalization and does not carry any "experimental" information, maybe skip it? If the experimental time resolution would be better will there still be a step or will it be smeared out? Can you give the values of the horizontal dashed lines (fitting) of the four and discuss their differences? How are the experimental times are defined. I would have thought that the time taken is at central times of delta t, this there would be none at $\omega t = 0, \omega t = 2\pi$ and probably at $\omega t = \pi$ neither? See also last paragraph before data availability statement (page 6). Is $(\omega \delta t)$ so small? Why don't you plot all data points measured or are those 20 points the only ones measured? In this case what is $N = 5 \times 10^4$ means? I could not find omega and δt (only 266.7 ns laser pulse and 1550 nm).

OUR REPLY

We thank the referee for these detailed inquiries regarding the data presentation and experimental parameters. We address them point-by-point below:

(1) Point at $\omega t = 0$: The first point at $\omega t = 0$ is indeed experimentally measured, not just normalized. We perform the state preparation and immediately measure the polarization state. This point is crucial as it serves as a calibration baseline, quantifying the fidelity of the initial state preparation. It is close to 1, which reflects the effectiveness of the state preparation.

(2) Time Resolution: The "step" observed in Fig. 2 (the abrupt jump from population $+1$ to -1) is not due to limited time resolution, but rather a consequence of the basis tracking convention. The abrupt change of the eigenstate observed in Fig. 2 occurs at the location of the diabolic line shown in Fig. 1(a). Importantly, this change does not signal any dynamical singularity. Instead, it arises from the smooth evolution of the system through a diabolic

degeneracy, where the instantaneous eigenstates interchange their ordering. Within the labeling convention adopted throughout the manuscript (see the discussion below Eq. (5) in the revised version), the eigenvalues are indexed such that $E_1 \geq E_2$. It is this fixed ordering convention, rather than any physical discontinuity, that gives rise to the apparent abrupt switch at the diabolic line. Consequently, the state transfer occurs at $t = T/2$ due to the projection of the continuously evolving state onto the given instantaneous eigenstates. Though the state vector itself always evolves smoothly along the chosen trajectories in the parameter space, as supported by Fig. S3 in the revised Supplementary Material. Therefore, even with infinite time resolution, the plot of the relative population (projected onto these basis states) would still exhibit a sharp 'discontinuity' where the basis definition flips. It is a representational feature, not an experimental time resolution.

(3) Horizontal Dashed Lines (Fitting Values): The horizontal dashed lines represent the numerical fitting of the experimental data, indicating the relative eigenstate population. We have added the fitting values in the caption of Fig. 2 of the revised manuscript to make the paper more explicit. The deviation from unity and the minor asymmetry between the upper and lower bounds stem from hardware imperfections and noise, such as the finite extinction ratio of the PBS, slight miscalibration of the VPCs, and insertion loss differences between the two optical paths.

(4) Definition of Experimental Times and $N = 5 \times 10^4$: As addressed in our response above to the referee's Comment 1, here N refers to the number of numerical integration steps used in the theoretical calculation to obtain the target operator $U(t)$. Choosing a large N ensures that the theoretical model is strictly adiabatic. Then, we select 21 representative "target times" $(t_0, \ldots, t_{20})$ along the state trajectory. For a given target point t_q (e.g., $\omega t_q = \pi$), we numerically integrate the Schrödinger equation from $t = 0$ to $t_q = \pi/\omega$ (using $N = 5 \times 10^4$ steps and each discrete time step is t_q/N to obtain the cumulative operator $U(t_q)$. This operator is then physically implemented through a single pass of light in the optical setup. Therefore, we can target exact phases like $0, \pi, 2\pi$ precisely by configuring the experimental setup for that specific time range.

(5) Values of ω and δ_t: We acknowledge that these two parameters are not explicitly provided. Here $\omega = \pi/50000$ and $\delta_t = t/N$. In our simulation approach, ω is chosen to be sufficiently small, ensuring a slow dynamical encircling of an EP and the validity of the simulated evolution. We have added these definitions in the *Methods* section of the revised

manuscript (page 8, the 2nd column, the last paragraph).

REFEREE'S COMMENT 6

4) The precision of the experimental initial state is announced to be 0.05% meaning that the deviations observed would not be visible. Thus the explanation for this deviation cannot be the initial state?

OUR REPLY

We agree that this small uncertainty alone cannot account for the deviations observed in the dynamical evolution results. The observed deviations primarily stem from imperfections in the physical implementation of the evolution operator $U(t)$. These implementation imperfections include hardware limitations, measurement errors, and inevitable noise, such as the finite extinction ratio of the PBS, slight miscalibration of the VPCs, insertion loss differences between the two optical paths, and the shot noise and instrument precision in the final polarimeter measurement. In the revised manuscript, we have added the discussion to explicitly state that the deviations are dominated by "experimental imperfections in implementing the non-unitary evolution operator and measurement uncertainties," rather than attributing them entirely to the initial state preparation (page 8, the 2nd column, the 1st paragraph).

REFEREE'S COMMENT 7

Remarks for the Supplementary:

Why is Fig. S2 using a different order than in the other figures? The theoretical value of lower limit of the fidelity could be stated. Is it the Peterman factor. Is it the same as in Fig. S7 (maybe use the same y-axis label values)? The agreement of the experiment with the theory is not very good. In (a) red points I would even think a linear function would match better. Looks like the fidelity is more sensitive than the relative eigenstate populations. Can you explain this?

OUR REPLY

We thank the referee for the detailed examination of the Supplementary Material. We have addressed these points as follows:

(1) We apologize for the inconsistency in the layout. In the revised Supplementary Material, we have rearranged Fig. S2 (the figure is now presented as Fig. S3 in the revised Supplementary Material) to strictly follow the same order as in Fig. 2 and Fig. 3 of the main text.

(2) Theoretical Lower Limit and Petermann Factor: The observed lower bound in Fig. S2 (Fig. S3) in the original (revised) manuscript originates from the non-Hermitian phase in which the system is initialized, where the eigenmodes are inherently non-orthogonal. Based on our theoretical analysis and the curves, the theoretical minimum fidelity occurs at the beginning of the evolution cycle, with a value of $\sim 4/9$. We have added these discussions in the revised Supplementary Material (the section titled "*Fidelity Between Evolving and Initial States*") to clarify and explain that the fidelity lower bound is determined by this initial eigenstate overlap. This is the same for Fig. S7.

(3) Experiment-Theory Agreement and Sensitivity: We acknowledge that the experimental data points for fidelity exhibit larger scatter compared to the relative eigenstate populations. The referee is correct that fidelity is significantly more sensitive than the population metric, and we would like to clarify the physical and mathematical reasons for this difference:

The relative eigenstate population presented in our work is calculated within the biorthogonal basis. In non-Hermitian systems, the right eigenstates $\{|\psi_m\rangle\}$ are not mutually orthogonal, but they form a biorthogonal basis with the left eigenstates $\{|\eta_n\rangle\}$, satisfying the condition $\langle\eta_n|\psi_m\rangle = \delta_{mn}$. To extract the population experimentally, we project the measured state $|\psi_{\text{exp}}\rangle$ onto the left eigenvectors of the Hamiltonian (i.e., calculating overlaps like $\langle\eta_n|\psi_{\text{exp}}\rangle$). This biorthogonal projection effectively "filters out" the contribution from the unwanted mode due to the orthogonality condition $\langle\eta_n|\psi_m\rangle = 0$ $(m \neq n)$. Consequently, the population metric P is inherently robust and less sensitive to small imperfections in the evolved state, leading to better agreement between theory and experimental data.

In contrast, the state fidelity, in Fig. S3 of the revised manuscript, is defined as the direct overlap magnitude $F = |\langle\psi_k|\psi_{\text{exp}}\rangle|^2$ between the initial and experimentally evolving

right states. Consequently, it becomes highly sensitive to all forms of experimental noise, including global phase fluctuations and intensity drifts. The "linear-like" scatter observed in the red points of Fig. S3 reflects this hypersensitivity to experimental noise, rather than a fundamental deviation from the theoretical trend.

Therefore, given the inherent robustness and ease of visualization of the eigenstate population and related metrics formulated in the biorthogonal basis, many studies on non-Hermitian state transfer [e.g., Nature 605, 256–261 (2022); Phys. Rev. A 102, 040201(R) (2020); Phys. Rev. Lett. 128, 110402 (2022)] adopt the eigenstate population as a primary observable for assessing transfer quality. This also motivates our choice of a similar metric.

To make this point more explicit, we have further added these discussions in the revised Supplementary Material (the section titled "*Fidelity Between Evolving and Initial States*").

REFEREE'S COMMENT 8

I'm missing details on the time steps etc. to obtain Fig. S6 (and S8). Why is the value -1 is approached closest after the jump and not at 100?

OUR REPLY

We thank the referee for the careful observation. We note an apparent abrupt change in the instantaneous eigenstates at $t = T/2$ in Fig. 2 of the main text, which precisely corresponds to the moment the evolution trajectory adiabatically transfers from one energy surface to another. Therefore, the same behavior should also be observed in Fig. S6 in the original manuscript (Fig. S7 in the revised manuscript).

As shown in Fig. S7, we set the total roundtrip number $N = 100$, meaning the full encircling period $T = 2\pi/\omega$ is discretized into 100 segments. Each roundtrip corresponds to a time step $\delta_t = T/N$. According to the experimental scheme in Fig. S5, each complete loop's simulation represents single evolution $e^{-iH(t_n)\delta_t}$. The simulation in the fiber loop is achieved by sequentially applying the evolution operator $U_n = e^{-iH(t_n)\delta_t}$ ($n = 1$ to 100) for each round trip, and the signal is looped 100 times. These 100 discrete steps thereby dynamically and completely simulate the state transfer process around an EP over one full period T. As described above, the state transfer always occurs at $t = T/2$, which corresponds precisely to

a round-trip number of $N/2 = 50$. The discrepancy between the simulation and theoretical results stems from the accumulation of numerical accuracy errors. The same behavior can be observed in Fig. S8, where we set roundtrip number $N = 200$, thus the state exchange occurs earliest at $N/2 = 100$.

To clarify this point, we have added some discussions in the caption of Fig. S7 in the revised Supplementary Material to improve readability.

REFEREE'S COMMENT 9

Will it be possible to plot the polarization components of the measured states directly in the supplementary, without projection, thus the measured data is presented?

OUR REPLY

We thank the referee for this valuable suggestion. In the revised Supplementary Material, we have added a new figure (Fig. S9) in the new section titled "*Evolution of the Polarization Components in the Measured States*", which directly plots the Stokes vector components (S_1, S_2, S_3) of the experimentally measured states, thereby characterizing their polarization evolution.

***** REPORT OF THE REFEREE #3 *****

REFEREE'S COMMENT I

The manuscript "Observation of Restored Adiabatic State Transfer in Time-Modulated Non-Hermitian Systems" reports the first experimental demonstration of restored adiabatic state transfer in a non-Hermitian system. Traditionally, encircling an exceptional point leads to chiral mode switching caused by complex energy spectra. In this work, authors utilize a specific trajectory in parameter space that constrains the evolution operator to a purely real spectrum, thereby suppressing non-adiabatic transitions and enabling symmetric, adiabatic state transfer regardless of the path direction. Using a photonic polarization platform, they further demonstrate controllable switching between symmetric and chiral state transfer within the same experimental architecture. The work is experimental validation of recent theoretical predictions (Phys. Rev. Lett. 133.11 (2024): 113802).

OUR REPLY

We thank the referee for the careful reading of our manuscript and agree with the summary made by the referee.

REFEREE'S COMMENT II

The results are of broad interest to the non-Hermitian, photonics, and quantum control communities. I find the manuscript well within the scope of Nature Communications. I have following questions/comments for authors:

OUR REPLY

We thank the referee for this overall positive evaluation of our work. Below we address the comments point by point.

REFEREE'S COMMENT 1

1. Parameters x and y are introduced mathematically in Eq. (2) to define the trajectories. Since these parameters are central to the claim of restored adiabaticity, a clearer explanation of mapping $(x, y) \to (\epsilon, k, \kappa, \varepsilon)$ and some intuitive explanation beside this abstract parametrization would significantly improve overall clarity.

OUR REPLY

We thank the referee for this relevant remark.

First, to improve clarity and avoid potential confusion, we have renamed some system parameters used in the original manuscript. Specifically, the parameters k and ε have been relabeled as u and v, respectively.

Second, in the revised manuscript, we have added a new paragraph below the newly added Eq. (3), which further clarifies the mapping employed in our work. Namely:

Specifically, the system parameters in Eq. (3) define a two-dimensional (2D) hyperboloid embedded in the 4D parameter space (ϵ, u, v, κ) satisfying $\epsilon^2 + u^2 - v^2 - \kappa^2 = \alpha^2$ (see also Supplementary Material for an intuitive geometric visualization of this manifold). Importantly, by restricting the system parameters to such a 2D manifold within the full 4D parameter space, one can effectively reduce the NHH in Eq. (1), which generally exhibits complex eigenvalues, into a parameter regime where the spectrum becomes purely real. Owing to the hyperbolic embedding defined by Eq. (3), each point (x, y) uniquely determines a single point in (ϵ, u, v, κ) in the full parameter space, and hence a unique Hamiltonian. Therefore, the spectrum of the NHH H is well defined. Physically, this restriction enforces a balance between gain and loss that removes the imaginary part of the spectrum.

REFEREE'S COMMENT 2

2. Given the experimental nature of this work, the authors should provide a more explicit description of how the set of Hamiltonian parameters gain/loss ε, dissipative coupling κ, coherent coupling k and detuning ϵ are controlled in the experimental setup in the main text.

OUR REPLY

We thank the referee for this important question. We agree that clarifying the link between the theoretical Hamiltonian parameters $\{v, \epsilon, \kappa, u\}$ and the experimental controls is essential for the reproducibility and clarity of our work. In the experiment, these Hamiltonian parameters are not tuned directly as physical quantities, but are implemented through the decomposition of the target evolution operator $U(t) = R_1 L R_2$. In the revised manuscript (*Methods* section), we have added a detailed description of the experimental setup to further explain how the Hamiltonian parameters are realized in the experiment (page 7, the 2nd column, the last paragraph):

In our experiment, as mentioned above, we simulate the continuous system dynamics by appropriately constructing the corresponding operator $U(t)$, which effectively mimics the induced time evolution. This is achieved as follows. At the beginning, by restricting the system parameters to the 2D manifold within the full 4D parameter space, one reduces the NHH $H_0(t) \to H(t)$, which generally exhibits complex eigenvalues, into a parameter regime where the spectrum becomes purely real. For a given point on the trajectory, the Hamiltonian parameters $\{v(t), \epsilon(t), \kappa(t), u(t)\}$ are determined by $\{x(t), y(t)\}$. After that, we compute the target cumulative evolution operator $U(t)$, and decompose this target operator into the form $U(t) = R_1 L R_2$ via the singular value decomposition.

In practice, these four parameters $\{v(t), \epsilon(t), \kappa(t), u(t)\}$ collectively contribute to each element of the matrix $U(t)$. However, for coherence parameters (detuning ϵ, coherent coupling u), the unitary parts of the evolution, which mix the polarization modes (basis rotations), are mainly encoded in the rotation matrices R_1 and R_2. These are implemented by adjusting the waveplate angles in the two variable polarization controllers. For non-Hermitian parameters (gain/loss

v, dissipative coupling κ), the non-unitary feature is primarily encoded in the diagonal matrix L. This is physically implemented by controlling the gain/loss ratio between the two split optical paths using the variable optical attenuator and erbium-doped fiber amplifiers together with the optical delay line.

REFEREE'S COMMENT 3

3. The adiabatic restoration relies on a specifically designed trajectory in parameter space. How sensitive is the adiabaticity to small deviations from this trajectory? Does a slight departure immediately re-introduce chiral behavior?

OUR REPLY

We thank the referee for this important question. The breaking of pseudo-Hermiticity of the Hamiltonian H in Eq. (4) of the revised manuscript by small perturbations does not, by itself, immediately lead to chiral mode behavior.

Specifically, for sufficiently weak perturbations, for example, by adding a term $\Delta\sigma_z$ to the Hamiltonian, i.e., $H \to H + \Delta\sigma_z$, with $\Delta \ll 1$, the system dynamics remains effectively adiabatic, provided that the induced imaginary components of the spectrum remain small. Under these conditions, non-adiabatic transitions do not occur, and chiral behavior is not observed. Our simulations, shown in the newly added Figs. S10 and S11, reveal that the symmetric adiabatic state transfer persists when the perturbation is small ($\Delta \leq 0.03$). However, as the perturbation strength increases beyond this threshold ($\Delta \geq 0.05$), distinct chiral behavior emerges.

To further clarify this point, we have added a detailed analysis and discussion about the perturbation in a new section titled "*Perturbation Analysis of the Observed Adiabatic Mode Switching*" in the revised Supplementary Material. We have also further discussed the robustness with respect to trajectory parameters and provided detailed discussions in the new section titled "*Impact of Evolution Speed and Loop Parameters on Restored Adiabaticity and Robustness Analysis*" in the revised Supplementary Material. These demonstrations provide rigorous support for the claim that the experimentally observed "restored adiabaticity" is a robust physical phenomenon.

In addition, we have modified the text in the section titled "*Experimental Simulation of Adiabatic and Symmetric State Transfer*" to explicitly emphasize the robustness of the observed adiabatic state transfer against perturbations that break the pseudo-Hermiticity of H. In particular, a new paragraph has been added at the end of this section (page 5, the 2nd column, the 4th paragraph):

> We further emphasize that the observed adiabatic symmetric state transfer remains robust even when the NHH H in Eq. (4) is weakly perturbed so that its spectrum becomes complex, provided that the perturbations are sufficiently small to prevent the onset of NATs. Additional details and supporting analysis are provided in the Supplementary Material.

REFEREE'S COMMENT 4

> 4. As I understand, the adiabaticity criteria used in this work is the absence of non-adiabatic transitions. Are there any constraints on how fast the loop can be traversed? To be specific, I am referring to ω in Eq. (3).

OUR REPLY

We appreciate this important remark. In the protocol implemented in our work, no specific constraints are imposed on the angular velocity ω or, equivalently, on the winding period T. The symmetric state transfer is observed for any choice of ω, provided that the evolution is sufficiently slow to suppress fast diabatic effects during the dynamics.

To further substantiate this point, we have performed additional numerical simulations over a broad range of winding periods T. The corresponding results are presented in the new plots shown in Figs. S12–S17 in the revised Supplementary Material. As demonstrated there, the protocol exhibits robust symmetric state transfer in the adiabatic limit $T \to \infty$ or $\omega \to 0$, while for sufficiently small T the dynamics enters the diabatic regime.

REFEREE'S COMMENT 5

5. The data in Fig. 2 of the main text appear noticeably less noisy than the corresponding results shown in Fig. S2. Could the authors clarify the origin of this difference?

OUR REPLY

We thank the referee for the careful inspection of the data. We acknowledge that the experimental data points for fidelity exhibit larger scatter compared to the relative eigenstate populations. The referee is correct that fidelity is significantly more sensitive than the population metric, and we would like to clarify the physical and mathematical reasons for this difference:

The relative eigenstate population presented in our work is calculated within the biorthogonal basis. In non-Hermitian systems, the right eigenstates $\{|\psi_m\rangle\}$ are not mutually orthogonal, but they form a biorthogonal basis with the left eigenstates $\{|\eta_n\rangle\}$, satisfying the condition $\langle\eta_n|\psi_m\rangle = \delta_{mn}$. To extract the population experimentally, we project the measured state $|\psi_{\text{exp}}\rangle$ onto the left eigenvectors of the Hamiltonian (i.e., calculating overlaps like $\langle\eta_n|\psi_{\text{exp}}\rangle$). This biorthogonal projection effectively "filters out" the contribution from the unwanted mode due to the orthogonality condition $\langle\eta_n|\psi_m\rangle = 0$ $(m \neq n)$. Consequently, the population metric P is inherently robust and less sensitive to small imperfections in the evolved state, leading to better agreement between theory and experimental data.

In contrast, the state fidelity, in Fig. S3 of the revised manuscript, is defined as the direct overlap magnitude $F = |\langle\psi_k|\psi_{\text{exp}}\rangle|^2$ between the initial and experimentally evolving right states. Consequently, it becomes highly sensitive to all forms of experimental noise, including global phase fluctuations and intensity drifts. The scatter observed in Fig. S3 reflects this hypersensitivity to experimental noise, rather than a fundamental deviation from the theoretical predictions.

Therefore, given the inherent robustness and ease of visualization of the eigenstate population and related metrics formulated in the biorthogonal basis, many studies on non-Hermitian state transfer [e.g., Nature 605, 256–261 (2022); Phys. Rev. A 102, 040201(R) (2020); Phys. Rev. Lett. 128, 110402 (2022)] adopt the eigenstate population as a primary observable for assessing transfer quality. This also motivates our choice of a similar metric.

To make this point more explicit, we have further added these discussions in the revised

Supplementary Material (the section titled "*Fidelity between Evolving and Initial States*").

REFEREE'S COMMENT 6

6. Since the present implementation relies on classical coherent light and photodetection, a brief discussion on the prospects and potential challenges of extending this approach to a quantum regime, where quantum noise may become relevant, would be useful.

OUR REPLY

We thank the referee for this forward-looking suggestion. Extending our protocol to the quantum regime (e.g., using single photons) is feasible, some similar related work [Phys. Rev. Lett. 134, 146602 (2025); Laser Photonics Rev. 18, 2300794 (2023)] has achieved this. This is indeed a promising direction for realizing robust quantum gates, but it entails specific challenges regarding quantum noise and probability.

For classical coherent light, the amplifying device can be used to compensate for the loss in the optical path and provide a certain amount of gain. In the quantum regime, however, linear amplification inevitably introduces quantum noise. This noise would destroy the coherence of fragile quantum states and degrade fidelity. To circumvent this noise, the extension to the quantum regime would typically employ a passive strategy. Instead of implementing physical gain, one can implement relative gain by strictly using attenuation. This avoids the introduction of added quantum noise. The process is probabilistic. Not every photon will survive the evolution process. Therefore, the realization requires post-selection to filter out the photon loss events. Such a passive approach will inevitably increase the losses during the evolution process, which is clearly detrimental to practical applications.

To emphasize this point, we have added a paragraph in the newly created *Discussion* section of the revised manuscript (page 6, the 2nd column, the first paragraph):

Although the protocol presented here has been implemented using a classical optical platform, its extension to the quantum regime, for instance, using single photons, is in principle feasible [36, 37] and represents a promising direction toward the realization of robust quantum gates. In the classical setting with

coherent light, optical losses can be compensated by amplifying elements, which may also provide controlled gain along the propagation path. In contrast, in the quantum regime, linear amplification unavoidably introduces quantum noise, which degrades coherence and fidelity of fragile quantum states [48]. To circumvent this limitation, relative gain can instead be implemented using purely passive attenuation, thereby avoiding noise injection [49]. This approach is inherently probabilistic, as not all photons survive the evolution, and consequently requires post-selection to discard loss events. While such a passive strategy increases overall losses and limits practical scalability, it nevertheless provides a viable route for experimentally accessing the quantum regime without compromising state coherence.

REFEREE'S COMMENT 7

7. The manuscript states that the demonstrated symmetric adiabatic state transfer may enable applications in topological quantum computation and related quantum information platforms. It is further claimed in the Supplementary Material that this protocol has potential applications in quantum communication, quantum computing, and quantum networks. These claims are rather general. It would strengthen the manuscript if the authors could provide concrete examples of specific tasks which would benefit from demonstrated protocol, or how this mechanism could be incorporated into realistic photonic or quantum-information architectures.

OUR REPLY

We thank the referee for this useful suggestion. In the revised manuscript, we specify the relevant tasks where the proposed protocol can lead to substantial advancements. Specifically, we have refined the text in the *Conclusions* section (last paragraph) where it now reads:

These results open new avenues for non-Hermitian state-control protocols in both classical and quantum photonic platforms, with potential implications for

topological quantum computation [51–55] and related areas of classical and quantum information processing. In particular, the symmetric state-switching protocol demonstrated here in a minimal two-level photonic building block can, in principle, be scaled to implement controlled topological state permutations in multi-qubit systems [44], for example, in integrated photonic architectures or networks of coupled waveguides and/or cavities, where it could realize SWAP-type quantum gate operations [56]. The controlled implementation of symmetric group operations in the non-Hermitian qubit eigenspace may enable topological quantum-computation schemes, where resulting holonomies generated by encircling EPs can be exploited as robust logical operations [52, 57, 58].

REFEREE'S COMMENT 8

Minor comments:

8. The choice of notation in Eq. (1) could be improved. Several symbols (e.g., k and κ, ϵ and ε) are visually similar and may lead to confusion and mix-up. Moreover, 'k' is repeated in the eigenstate label $|\psi_k\rangle$ elsewhere in the text.

OUR REPLY

We thank the referee for pointing out these notational ambiguities. We agree that clear notation is essential for readability. We have carefully checked the symbols in Eq. (1) and the notation throughout the manuscript. To avoid confusion, we have changed the symbol k to u and replaced the symbol ε with v to distinguish it clearly from ϵ throughout the text in the revised manuscript and Supplementary Material.

REFEREE'S COMMENT 9

9. Do the colors in Fig. S1 represent a physical quantity, or are they included purely for aesthetic purposes?

OUR REPLY

The colors of the surfaces in Fig. S1 correspond to the default colormap of the visualization software and are included for graphical clarity only. They do not represent any physical quantity.

***** REPORT OF THE REFEREE #4 *****

REFEREES' COMMENT

I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

OUR REPLY

We thank the referee for the time spent to review our manuscript.

***** REPORT OF THE REFEREE #5 *****

REFEREE'S COMMENT I

In this work, the authors simulated adiabatic state transfer by encircling an exceptional point (EP) in a non-Hermitian system using the two polarizations of a laser. Their approach entails simulating a sequence of individual non-unitary adiabatic evolution operators, $U(t_n)$, for discrete time slices t_n. By decomposing $U(t_n)$ as $R_1 L R_2$, the unitary operations R_1 and R_2 can be realized using a standard wave plate setup, while the diagonal matrix L can be implemented using components such as an erbium-doped fiber amplifier and optical delay lines.

OUR REPLY

We thank the referee for the time devoted to reviewing our manuscript. We agree with the summary made by the referee. Below, we address step-by-step all the comments raised by the referee.

REFEREE'S COMMENT 1

However, this approach does not seem to actually realize a full adiabatic state transfer process, as it simulates only a sequence of discrete, individual non-unitary evolution operators, $U(t_n)$.

OUR REPLY

We thank the referee for this comment and would like to clarify the point of potential confusion.

In our work, the continuous state dynamics is simulated by directly constructing a single non-unitary operator $U(t)$, which represents the cumulative time evolution of the system over the interval $[0, t]$, as detailed in the *Results* section below Eq. (6) in the revised manuscript. This operator is implemented experimentally in a single pass through our optical setup, thus

effectively emulating the continuous evolution of the state up to the chosen time t.

Importantly, our approach does not rely on a stroboscopic implementation, in which the dynamics is approximated by a sequence of discrete Hamiltonians with incrementally varied parameters and each experimental data point corresponds to a separate static Hamiltonian realization, as seen, for example, in [Phys. Rev. Lett. 86, 787 (2001)] (cited as Ref. [21] in the revised manuscript).

Instead, our setup enables the faithful simulation of the continuous dynamics for any evolution time t. Specifically, the 21 representative "target times" $(t_0, \ldots, t_{20})$ used in the experiment correspond to 21 independently constructed evolution operators $U(t_k)$. For a given target time t_k (e.g., $\omega t_k = \pi$), we numerically integrate the Schrödinger equation from $t = 0$ to $t = t_k$ using $N = 5 \times 10^4$ integration steps, thus obtaining the cumulative operator $U(t_k)$. This operator is then applied to the initial state in a single experimental run.

In this way, specific evolution phases (such as $0, \pi, 2\pi$) can be directly targeted and realized by appropriately configuring the optical setup for the corresponding operator $U(t_k)$, rather than by incrementally stepping through intermediate parameter values of the Hamiltonian, as is done in Ref. [21].

To further highlight the principle of our experimental setup and dispel any confusion, we have added new paragraphs in the revised manuscript (page 4, the 2nd column, the 2nd paragraph):

> For a given initial polarization state $|\Psi_0\rangle$, the system evolution is described by
>
> $$|\Psi(t)\rangle = U(t)|\Psi_0\rangle,$$
>
> where $|\Psi(t)\rangle = [a(t), b(t)]^T$ represents the polarization state at evolution time t, with $a(t)$ and $b(t)$ denoting the horizontal and vertical field components, respectively.
>
> In the experiment, we simulate the continuous system dynamics, generated by a time-dependent NHH $H(t)$, by constructing a family of 'static' non-unitary evolution operators $U(t)$, each of which corresponds to the time evolution from $t = 0$ to a chosen target time t. Specifically, in the experiment, we select 21 representative target times $(t_0, \ldots, t_{20})$. Each target time t_q corresponds to an independently constructed operator $U(t_q)$. For example, for $\omega t_q = \pi$, the operator $U(t_q)$ represents the evolution from $t = 0$ to $t_q = \pi/\omega$. This operator is then

applied to the initial state in a single experimental run (as shown in Fig. 1). As such, hereafter, unless stated otherwise, the terms 'time evolution' and 'system dynamics' refer exactly to this simulated evolution.

To obtain each $U(t_q)$, we first numerically integrate the Schrödinger equation from $t = 0$ to $t = t_q$. This integration is performed using $N = 5 \times 10^4$ time steps, which ensures the convergence of the time-ordered exponential (see also Supplementary Material). Formally, the evolution operator is approximated as

$$U(t_q) \approx \prod_{n=1}^{N} \exp[-iH(t_n)\,\delta t]\,,$$

where $\delta t = t_q/N$ and $t_n = (n - \frac{1}{2})\delta t$ denotes the midpoint of each integration interval. Each such numerically constructed operator $U(t_q)$ is then physically implemented by appropriately configuring the optical elements, thereby directly mapping the integrated evolution onto the output state. Repeating this procedure for different target times t_q allows us to reconstruct the continuous dynamics of the system without relying on incremental or stroboscopic parameter modulation.

We emphasize that our experiment is a proof-of-principle demonstration, designed to clearly reveal the novel physical mechanism underlying adiabatic state transfer. By effectively simulating the continuous dynamics using the constructed non-unitary operators $U(t)$, the protocol allows us to avoid error accumulation that would arise in a genuine continuous multi-step experimental implementation of the same dynamics. In this way, our work provides the first experimental demonstration that adiabaticity can be restored in the dynamics of non-Hermitian systems when encircling exceptional points.

REFEREE'S COMMENT 2

That said, the ability to experimentally access an EP-encircling trajectory is still of high potential scientific value. Given the substantial body of work on state transfer by encircling EPs in non-Hermitian systems [e.g., Phys. Rev. Lett. 86, 787 (2001); Nature 537, 76 (2016)], the authors should clearly articulate the significance and novelty of this work i.e. what distinguishes their approach

from previous implementations or whether they have overcome specific technical challenges.

OUR REPLY

We thank the referee for this comment and address the point below by clarifying the relation of our work to previous studies on EP-encircling protocols.

As emphasized in the revised *Introduction*, the central achievement of our work is the *first* demonstration of *adiabatic*, *symmetric*, and *fully dynamical* state transfer achieved by encircling an exceptional point. The simultaneous realization of these three ingredients—**adiabaticity**, **symmetry**, and **continuous dynamics**—distinguishes our approach from all previous theoretical and experimental implementations of EP-encircling.

To place our results in context, prior studies of *dynamical* EP-encircling, both theoretical [38–40] and experimental [27–32, 35, 36, 41] (reference numbers as in the revised manuscript), have consistently reported chiral state transfer in the adiabatic, i.e., dynamically slow, limit. In these works, the presence of a complex energy spectrum leads to non-adiabatic transitions (NATs) during the evolution: the system dynamically selects the instantaneous eigenstate with lower loss, resulting in a direction-dependent final state that is independent of the initial condition. Consequently, adiabatic EP encirclement has been widely regarded as intrinsically linked to chiral, direction-dependent dynamics, making symmetric state transfer appear fundamentally inaccessible under continuous evolution.

The work of Phys. Rev. Lett. 86, 787 (2001) [Ref. [21] in the revised manuscript] indeed reported symmetric state exchange associated with EP-encircling; however, this is achieved in a quasistatic (stroboscopic) manner. Similar theoretical [19, 20] and experimental [22–26] works are also cited in the revised manuscript. In Ref. [21], the system parameters are incrementally changed and the state is measured independently at each step, without realizing genuine *continuous* dynamics. As such, this approach does not probe the *dynamical* evolution of the system during the encircling process.

In contrast, Ref. [Nature 537, 76 (2016); Ref. [27] in the revised manuscript] realized *continuous dynamical* encircling of an EP, but necessarily observed *chiral* state transfer, in agreement with the previous prevailing understanding. The unavoidable imaginary spectrum in that system induced NATs, which broke adiabaticity and thus enforced directional

asymmetry.

Our work overcomes this apparent dichotomy. By engineering a dynamical EP-encircling protocol, where the evolving state remains on real-valued Riemann sheets of the spectrum, we suppress NATs and dissipation-driven instabilities while maintaining continuous time evolution. This enables, for the first time, a genuinely dynamical and adiabatic EP-encircling protocol that produces symmetric, direction-independent state transfer, with the final state determined solely by the initial condition.

In summary, our key contribution is the experimental realization of a regime previously thought unattainable: the coexistence of adiabatic dynamical EP encirclement and symmetric state switching within a single physical platform. We believe this resolves a previously prevailing conceptual limitation in non-Hermitian dynamics and establishes a new paradigm for controlled state manipulation using EPs.

REFEREE'S COMMENT 3

The manuscript repeatedly states that the evolution operator acquires a purely real spectrum along the designed EP-encircling trajectories. It would be good to clearly explain the necessity of this. Since the same photon does not actually undergo repeated $U(t)$ slice evolutions, some loss is ok too?

OUR REPLY

We thank the referee for this question. The issue raised concerns the core physical mechanism of our protocol, which is already addressed in the manuscript, as we clarify below.

The necessity of a purely real spectrum along the EP-encircling trajectories is directly connected to the suppression of NATs and, consequently, to the restoration of continuous adiabatic evolution in time-modulated non-Hermitian systems.

As already emphasized in the *Introduction* of the original manuscript, it is precisely the imaginary part of the spectrum in generic non-Hermitian Hamiltonians that gives rise to NATs during dynamical EP-encircling. These transitions occur because eigenstates associated with larger imaginary parts of the eigenvalues are dynamically favored, leading to

discontinuous state jumps and, ultimately, to chiral, direction-dependent state transfer. In this sense, the emergence of NATs and the breaking of directional symmetry are intrinsically linked to the presence of complex-valued spectra.

By contrast, when the evolution operator exhibits a purely real spectrum along the encircling trajectory, this imbalance between gain and loss is eliminated. As a result, the system no longer experiences preferential population of a particular eigenstate, NATs are suppressed, and a smooth adiabatic evolution becomes possible. This is the key reason why real-valued spectra are essential for achieving symmetric, direction-independent state transfer under continuous dynamical encircling of an EP.

To make this point more explicit, we have further clarified the discussion in the Introduction of the revised manuscript. In particular, we now write (page 1, the 2nd column, the 1st paragraph):

> When the same EP-encircling is implemented *dynamically*, however, qualitatively different behavior emerges. A prominent consequence is chiral mode transfer: when dynamically encircling an EP, the final state depends solely on the encircling direction regardless of the initial state [27–37]. The complex Riemann surface structure in the system's spectrum underlies this chiral behavior, namely, the eigenstate with the larger imaginary part of the eigenvalue is preferentially populated due to non-adiabatic transitions (NATs) which occur during the state evolution [27].

Furthermore, in the subsequent paragraph we explicitly connect the restoration of adiabaticity to the real-valued nature of the spectrum:

> A recent theoretical study [42] has demonstrated that adiabatic and symmetric state transfer can be restored in non-Hermitian systems. In this context, the restored adiabaticity refers to the absence of discontinuous NATs during the state evolution. Such behavior is realized by dynamically encircling an EP along suitably designed trajectories in parameter space, for which the evolution operator exhibits a purely real spectrum. The resulting suppression of NATs thus enables smooth and symmetric state transfer. This behavior is in sharp contrast to previously studied dynamical EP-encircling protocols, where complex

spectra generically trigger non-adiabatic jumps and give rise to chiral, direction-dependent state transfer [27,28].

We believe these clarifications directly address the referee's concern by explicitly explaining why enforcing a real-valued spectrum is not merely a technical detail, but a necessary physical condition for suppressing NATs and enabling the observed symmetric adiabatic state transfer.

Concerning the second part of the question, additional losses are admissible provided they do not induce a non-negligible imaginary component in the spectrum, which would otherwise trigger non-adiabatic transitions and consequently break the continuous and symmetric state evolution. We provide further details on this point in our response to the following comment.

REFEREE'S COMMENT 4

The use of 21 discrete time points is sufficient for the purpose of the main figures, i.e., to identify the direction-independent final state after a full encircling cycle, since the reported $P(t)$ dynamics is largely piecewise constant with a clear switching outcome. However, the more critical issue for the strong claim of "restored adiabaticity" is not the temporal sampling density, but the lack of a systematic validation with respect to control parameters and numerical/experimental discretization. In particular, since the time-ordered evolution is approximated as a product over N segments (with $N = 5 \cdot 10^4$ chosen in the experiment), the authors should demonstrate that the reported state-transfer outcome converges with respect to N, and that the adiabaticity metrics improve as the protocol is slowed (e.g., by scanning T or omega), for both encircling directions. Robustness checks against loop parameters would further strengthen the generality of the "restoration" claim. Since this may require substantial additional experimental efforts, a numerical demonstration would also suffice.

OUR REPLY

We thank the referee for this constructive suggestion. We fully agree that validating the convergence with respect to the discretization steps (N) and verifying the adiabatic

behavior with respect to the evolution speed (ω or T) are essential to substantiate the claim of "restored adiabaticity." As suggested, we have performed systematic numerical simulations to address these points. The new results are included in new Fig. S12–S19 and the new section titled "*Impact of Evolution Speed and Loop Parameters on Restored Adiabaticity and Robustness Analysis*" of the revised Supplementary Material.

Firstly, with the evolution speed set to $\omega = \pi/50000$, we have simulated the state transfer fidelity as a function of the total number of discretization steps N, ranging from 10 to 10^4. The simulation results confirm that the relative eigenstate population $P(t)$ converges rapidly to unity as N increases. Specifically, the deviations in $P(t)$ become negligible starting from $N = 1000$. Thus, our choice of $N = 5 \times 10^4$, used to obtain the non-unitary operator $U(t)$, ensures that the generated state evolution is in good agreement with theoretical predictions, confirming that our experimental implementation accurately captures the underlying adiabatic dynamics. Conversely, in the low-N regime, the adiabatic behavior degrades significantly.

Next, to investigate the impact of the evolution speed ω (where the total evolution time is $T = 2\pi/\omega$) on the adiabatic behavior, we have simulated $P(t)$ with various ω for some fixed values of N. We observe that the evolution speed plays a critical role in maintaining adiabaticity: a slower evolution speed leads to more pronounced adiabatic behavior. Notably, even with a large N, the non-adiabaticity remains suppressed if ω is excessively large. Unlike the chiral encircling of an EP – which occurs due to non-adiabatic transitions – the observed restored symmetric transfer here is ensured by the adiabatic evolution ($\omega \to 0$).

We also numerically evaluate the robustness of the scheme by varying the loop parameters, which control the "size" (i.e., the center and radius) of the trajectory in parameter space. The results demonstrate that as long as the trajectory remains on the real-spectrum hyperboloid manifold and encircles the EP, the symmetric adiabatic transfer is preserved. This highlights the topological protection inherent in our scheme.

We further confirm that these behaviors hold for both clockwise and counterclockwise directions, reinforcing the inherent symmetry of the protocol. These numerical demonstrations provide rigorous support for the claim that the experimentally observed "restored adiabaticity" is a robust physical phenomenon. We have included these figures and detailed discussions in the new section titled "*Impact of Evolution Speed and Loop Parameters on Restored Adiabaticity and Robustness Analysis*" of the revised Supplementary Material.

REFEREE'S COMMENT 5

The manuscript repeatedly emphasizes a "genuine programmable symmetric–chiral (symmetric–asymmetric) two-mode switch," "long thought unattainable" and "universal." However, the demonstrated "switching" is realized by implementing two distinct effective Hamiltonians, H in Eq. (2) and H_0 in Eq. (4), within the same optical platform, with the two models coinciding only at $t = 0$ by construction for comparability. Please clarify in what precise sense the switch is "programmable" and "universal": is there a single continuous control knob within a single Hamiltonian family that tunes the system from symmetric/adiabatic to chiral/non-adiabatic transfer, or is the programmability mainly due to the platform's ability to emulate different models? If the former is intended, it would be important to provide evidence of a continuous crossover (e.g., by gradually introducing the imaginary-part spectrum or controlled deviations from the constraint manifold) rather than presenting two separate model realizations.

OUR REPLY

We thank the referee for this insightful comment, which allows us to clarify the meaning of "programmable" and "universal" as used in the manuscript.

By programmable, we do not mean that the system is tuned between symmetric (adiabatic) and chiral (non-adiabatic) state transfer via a single continuous control knob within a fixed effective Hamiltonian. Rather, programmability refers to the ability of the same physical platform to realize different dynamical regimes through an appropriate choice of the time-dependent modulation functions of the system parameters.

More specifically, our starting point is a generic non-Hermitian two-mode Hamiltonian H_0 defined on a four-dimensional parameter space as defined in Eq. (1). By selecting different closed trajectories in this parameter space, implemented experimentally via different parameter modulation protocols, the system can be programmed to exhibit either: (i) symmetric, adiabatic state transfer [via parameter modulations given in Eqs. (3), (6) and (4)], or (ii) chiral, direction-dependent state transfer [via parameter modulations given in Eqs. (3), (6)

and (10)].

Importantly, no hardware reconfiguration is required to switch between these regimes. The distinction arises solely from how the same set of control parameters is dynamically modulated in time. In this sense, the switching functionality is programmable at the level of control EP-encircling protocols rather than by altering the physical system.

Namely, the symmetric and chiral regimes correspond to dynamically encircling an EP either on or off a specific constraint manifold [hyperboloid surface, as given in Eq. (3)] on which the spectrum remains purely real. Remaining on this hyperboloid manifold suppresses NATs and enables symmetric state transfer, while departing from it generically introduces an imaginary spectrum and leads to NAT-induced chiral dynamics. Because the onset of NATs is tied to the emergence of imaginary eigenvalues, a smooth continuous crossover between the two regimes is not generically expected, and the use of distinct modulation protocols reflects this physical distinction rather than an artificial separation of models.

Regarding the term "universal", we use it in the restricted sense that the protocol applies to generic two-mode non-Hermitian systems with exceptional points, independent of the specific physical realization.

To further clarify the scope of the term "programmable" used, we have added the following two sentences in the *Introduction* section (page 2, the 1st column, the 3rd paragraph):

> This constitutes the first realization of a programmable symmetric–chiral state switch, a capability that was previously considered hard to achieve in non-Hermitian two-level systems. Specifically, for a given initial and final Hamiltonian, the nature of the state transfer (symmetric or asymmetric) can be selected solely through the proper choice of parameter modulation within a single physical platform.

***** REPORT OF THE REFEREE #6 *****

REFEREES' COMMENT

I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

OUR REPLY

We thank the referee for the time spent to review our manuscript.

A list of changes and summary

1. Text in both the main manuscript and Supplementary Material has been substantially revised and clarified.

2. A new section *Discussion* has been added in the main manuscript.

3. New figures, namely Fig. S2 and Figs. S9–S19, have been added in the revised Supplementary Material.

4. New sections *Dynamical Mapping via Discretized Unitary Decomposition*, *Evolution of the Polarization Components in the Measured States*, *Perturbation Analysis of the Observed Adiabatic Mode Switching*, and *Impact of Evolution Speed and Loop Parameters on Restored Adiabaticity and Robustness Analysis* have been added in the revised Supplementary Material.

5. Equations (1) and (2), along with subsequent mathematical expressions, have been updated to reflect the relabeling of the system parameters, specifically $k \to u$ and $\varepsilon \to v$.

All the changes and modifications in the revised version of the manuscript and supplementary are highlighted in red text in the additionally submitted file.

In summary, we sincerely thank the referees for their careful and constructive comments. We have thoroughly revised the manuscript to address all concerns and believe that this revised version is now suitable for further consideration in *Nature Communications*.

***** REPORT OF THE REFEREE 1 *****

REFEREE'S COMMENT I

The authors have carefully addressed not only my previous comments but also those of the other reviewers. Their detailed responses improve the overall clarity of the manuscript, and the experimental realization of the proposed phenomenon is timely and interesting.

OUR REPLY

We thank the referee for this positive summary of our work and for dedicating time to the thorough review of our manuscript.

REFEREE'S COMMENT II

After reviewing the revised manuscript, supplementary material, and rebuttal, I am left with one remaining conceptual issue that should be clarified. While the experimental realization and practical aspects of the work are compelling, the remaining issue concerns the conceptual and formal consistency of the underlying Hamiltonian description.

The central claim of the work is that "*we report the first experimental demonstration that the adiabatic state evolution can be fully restored when dynamically encircling an EP, regardless of the winding direction.*" This is an important statement, particularly in relation to previous studies on EP encircling and non-adiabatic state switching.

In this context, there is a fundamental ambiguity in the Hamiltonian formulation. According to Eqs. (3)–(5), the EP is identified at the condition $\alpha = 0$. However, at the same point, ϕ is not well-defined, and the present parametrization becomes ill-defined. As a result, the Hamiltonian in Eq. (4), which is expressed in terms of α and ϕ, involves a product of vanishing and diverging quantities, making it unclear whether the Hamiltonian is well-defined exactly at the EP.

If the Hamiltonian is not well-defined at that point, it becomes difficult to rigorously establish that the point itself is an exceptional point of the model. For example, key EP properties such as eigenvector coalescence and self-orthogonality,

$$\langle \eta_{\mathrm{EP}} | \psi_{\mathrm{EP}} \rangle = 0, \tag{1}$$

are not explicitly demonstrated in a well-defined manner within the current parametrization.

At the same time, the dynamical phenomena observed when encircling the region are likely consistent with known EP-related behavior. This mismatch between the ill-defined Hamiltonian at the EP and the observed EP-like behavior creates a conceptual ambiguity: if the Hamiltonian is not well-defined at the EP itself, it becomes unclear whether the system encircles a well-defined EP.

This issue could be addressed in several ways. For example:

1. The authors could provide an equivalent Hamiltonian representation that remains well-defined at the EP, and reformulate their analysis accordingly.

2. Alternatively, they could explicitly clarify that the present parametrization becomes ill-defined at the EP, and explain why the observed dynamics nevertheless reproduces EP-like behavior when encircling the region. In particular, the origin of the EP-like topology and state evolution in the absence of a well-defined Hamiltonian at the EP should be clearly articulated.

I believe that this issue should be addressed before the manuscript can be considered for publication.

OUR REPLY

We thank the referee for raising this important point concerning the behavior of the Hamiltonian at the exceptional point (EP). In the revised manuscript, we address this issue following the referee's suggestion (option 1) by providing additional analysis of the original Hamiltonian representation H_0 that remains well-defined at the EP.

In the main text, we introduced a transformed non-Hermitian Hamiltonian $H_0 \to H$ obtained by mapping the system parameters onto the hyperbolic surface via Eq. (3). While

this parametrization is continuous over the parameter space, it becomes singular at the EP, where the original Hamiltonian H_0 becomes defective. As a result, the parametrization in terms of α and ϕ involves formally vanishing and diverging quantities, which nevertheless give finite values for the system parameters and result in the finite and well-defined eigenvalue spectrum of the NHH H. However, the eigenvectors of the transformed Hamiltonian H, given in Eqs. (11) and (12), appear to diverge when approaching the EP.

To resolve this apparent ambiguity concerning the EP in the spectrum of H, in the newly added section 2 in the Supplementary Material, we analyze the system in the original representation, where the Hamiltonian H_0 [Eq. (1) in the main text] is well-defined over the entire parameter space, including the EP. By tracking the behavior of the corresponding right eigenvectors $|\psi\rangle$ and left eigenvectors $|\eta\rangle$ of H_0 as functions of the parameters (x, y), we explicitly verify the defining properties of an EP. More specifically, we first calculate the eigenspectrum of H_0 as a function of the parameters (ϵ, u, v, κ), and then apply the hyperbolic map in Eq. (3) in the main text.

In particular, we confirm that the point $(x = 0, y = 1)$ corresponds to an EP: (i) the biorthogonal norm vanishes, $\langle \eta_i^{EP} | \psi_i^{EP} \rangle = 0$, indicating self-orthogonality, and (ii) the right eigenvectors coalesce, as quantified by $\langle \psi_1^{EP} | \psi_2^{EP} \rangle = 1$. These features are explicitly demonstrated in the revised Supplementary [newly added Sec. 2 and Fig. S2(a,b)], and the newly added figure is also shown below as Fig. 1.

Therefore, the EP is well-defined in the original Hamiltonian H_0, and the apparent singularity arises solely from the choice of parametrization in the transformed Hamiltonian H. In this regard, we have also added a new sentence in the main manuscript highlighting the breakdown of continuity of the applied hyperbolic map at the EP. Specifically, the modified text in the revised manuscript (see page 2, the 2nd column, the last paragraph) reads:

> More precisely, the representation of this hyperbolic mapping via Eq. (4) is one-to-one and continuous throughout the parameter space except at an EP, where the parametrization becomes singular due to the defectiveness of the Hamiltonian H_0. This singularity, however, reflects a breakdown of the coordinate representation rather than of the Hamiltonian itself, which remains well-defined in the original parametrization (see below).

And the text in the paragraph below:

Notably, the eigenvalues of the NHH H remain well-defined at the EP ($\alpha_{EP} = 0$), while the eigenvectors given in Eqs. (11) and (12) become singular ($\phi_{EP} \to -i\infty$). This behavior is a direct manifestation of eigenvector coalescence in the original representation of the NHH H_0 in Eq. (1) (see Supplementary Material for details). We emphasize that, in our protocol, the EP is only encircled and never crossed. Consequently, the eigenvectors of H and their evolution along the winding trajectories remain well-defined throughout the process.

And the newly added figure in the revised Supplementary is shown below:

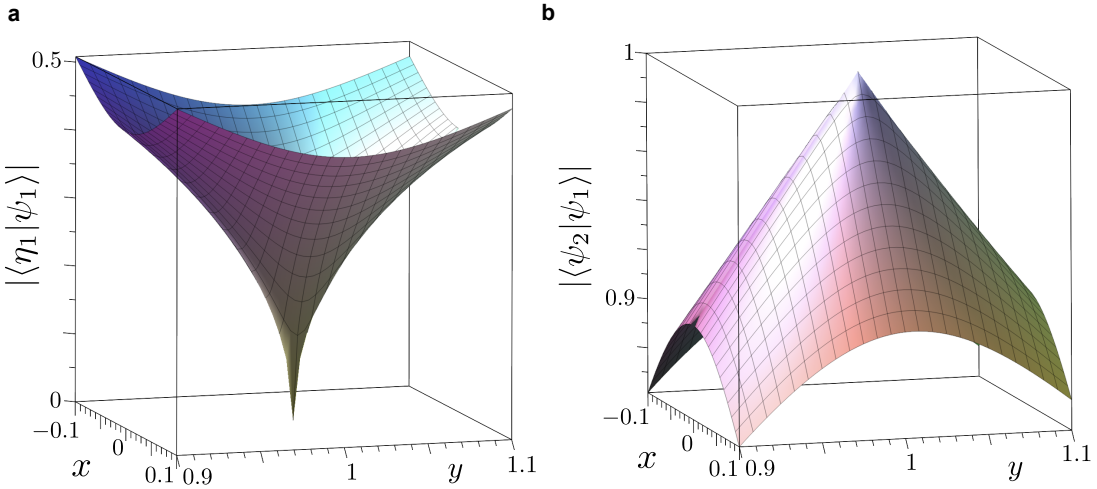

FIG. 1: Eigenvectors overlap of the non-Hermitian Hamiltonian H_0 [Eq. (1) in the main text], demonstrating the characteristic signatures of an exceptional point (EP) in the (x, y) parameter space. **a** Biorthogonal overlap between left and right eigenvectors, which vanishes at the EP located at $(x = 0, y = 1)$, indicating self-orthogonality. **b** Overlap between the two right eigenvectors of H_0, which approaches unity at the same point, signaling the coalescence of eigenvectors at the EP.

***** REPORT OF THE REFEREE #2 *****

REFEREE'S COMMENT I

Second report for 'Observation of Restored Adiabatic State Transfer in Time-Modulated Non-Hermitian Systems' by Xiaowei Wang, Ievgen I. Arkhipov, Quan Lin, Huixia Gao, Dengke Qu, Lei Xiao, Franco Nori, and Peng Xue (NCOMMS-25-90175).

The authors have well replied to my remarks (and the ones from the other referees) and I think deserves to be published in Nature Communications but would like that it is stressed that it is not an "in-situ" encircling.

OUR REPLY

We appreciate the referee's positive assessment of our previous revision and for additional helpful suggestions. Our point-by-point responses and the associated manuscript revisions are detailed below.

REFEREE'S COMMENT II

Still I'm a bit confused about the way the encircling is performed "experimentally". I think it should be clearly stated that the experiment is not an "in-situ" encircling but is done via numerically multiplying experimental results. The authors detail how the U operator is adjusted using numerical simulations but the statement that at the end 21(?) measurements are done giving $\psi_i^{\rm exp}$ measured states (with averaging?) and the final state is generated by multiplying this states. Is this correct?

OUR REPLY

We thank the referee for this comment, which allows us to further clarify the implementation of our simulation. We acknowledge that our implementation is not an "in-situ" encircling, but rather a simulation of such a process through "static" non-unitary evolution

operators $U(t)$, which we have already clarified in the previous version of the manuscript. To prevent any further ambiguity, we have explicitly added the description in the revised manuscript by changing the expression "As such, hereafter, unless stated otherwise, the terms 'time evolution' and 'system dynamics' refer exactly to this simulated evolution." into "As such, hereafter, unless stated otherwise, the terms 'time evolution', 'dynamically encircling', and 'system dynamics' refer experimentally to this simulated evolution, rather than to a genuine 'in-situ' encircling." to state that our work constitutes a discretized mapping of the adiabatic trajectory rather than an "in-situ" continuous encircling (see page 4, the 2nd column, the 4th paragraph).

There appears to be a slight misunderstanding regarding our approach. For each measurement time t_q, the evolution operator $U(t_q)$ is independently calculated according to Eq. (8) in the main text. Specifically, $U(t_q)$ represents the total evolution from the initial time $t = 0$ to the specific time t_q. By applying this operator $U(t_q)$ directly to the initial state as described in Eq. (7), we simulate the evolution process at that particular moment through a single experimental run. This procedure is performed independently for all 21 selected time points, including the final state at t_{21}. It is important to emphasize that each evolution process is characterized independently; the final state is not generated by the sequential multiplication of intermediate states. This method ensures that we accurately characterize the state evolution process at discrete intervals without accumulating errors from the step-by-step multiplication in experiments.

REFEREE'S COMMENT III

> Concerning the error bars are they obtained using the information of several experimental runs (i.e. standard deviation of N_{exp} samples, then give numbers) or obtained by estimating errors by precision of the elements?

OUR REPLY

We apologize for the confusion caused by our previous description regarding the error bars. We would like to clarify that these error bars are derived directly from the experimental data and represent the standard deviation of the measurement results. Specifically, each

experimental data point presented in the figures is obtained from 2560 samples. It is not obtained by estimating errors by precision of the elements. We have already clarified the physical meaning of the error bars in the caption of Fig. 2.

REFEREE'S COMMENT IV

> The statement in the conclusion "To the best of our knowledge, we report the first experimental demonstration that the adiabatic state evolution can be fully restored when dynamically encircling an EP, regardless of the winding direction." should be modified (don't know whether first experimental is valid in nature communications anyway) as this implies that the experiment is performing the dynamically encircling "in-situ" which it does not! Thus a statement would be more like: we report experimental evidence that ... by measuring the individual linearized evolution and generate the dynamical encircling in the computer by multiplying the experimental results ...

OUR REPLY

We thank the referee for raising this point. In the revised manuscript, following the referee's suggestion, we further clarify that the dynamics in our experimental protocol is not implemented *in situ*, but rather realized through an effective simulation within our optical setup. Specifically, in Sec. *Conclusions*, we revised the first two sentences as (on page 7, the 1st column):

> In this work, we report experimental evidence that adiabatic state evolution is fully restored during an EP-encircling protocol, which is realized through a sequence of individual evolution operators that scan the parameter space.

***** REPORT OF THE REFEREE #3 *****

REFEREE'S COMMENT

The authors have addressed the referee concerns. At this point, I recommend publication in Nature Communications.

OUR REPLY

We thank the referee for the positive review and recommendation.

***** REPORT OF THE REFEREE #4 *****

REFEREES' COMMENT

I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

OUR REPLY

We thank the referee for the time spent reviewing our manuscript.

*** REPORT OF THE REFEREE #5 ***

REFEREE'S COMMENT I

The revised manuscript has been significantly improved and the main concerns raised previously have been adequately addressed. The additional discussions in the revised manuscript and Supplementary Material make the physical interpretation much clearer. I therefore support publication of this work.

OUR REPLY

We thank the referee for the time devoted to reviewing our manuscript and for the positive assessment.

REFEREE'S COMMENT II

I only have one minor comment. Since the term "adiabatic" may still invite the conventional Berry-Kato interpretation, the authors may consider adding one brief reminder that here it is used in the operational sense of the absence of discontinuous non-adiabatic transitions.

OUR REPLY

We thank the referee for this comment and would like to clarify the point of potential confusion.

As suggested, we have already clarified this distinction in our previous response to Referee 1. Furthermore, we have incorporated a brief reminder to explicitly state that the term "adiabatic" is used here in the operational sense of the absence of discontinuous non-adiabatic transitions in the revised manuscript (see page 4, the 1st column, the 1st paragraph):

In particular, adiabaticity in the Berry–Kato sense, which is based on the following instantaneous eigenstate in the presence of a finite spectral gap, is not directly applicable here. The term "adiabatic" we use is in the operational sense of the absence of discontinuous non-adiabatic transitions.

***** REPORT OF THE REFEREE #6 *****

REFEREES' COMMENT

I co-reviewed this manuscript with one of the reviewers who provided the listed reports. This is part of the Nature Communications initiative to facilitate training in peer review and to provide appropriate recognition for Early Career Researchers who co-review manuscripts.

OUR REPLY

We thank the referee for the time spent reviewing our manuscript.

A list of changes and summary

1. In response to Referee #1's comments, we have clarified the parametrization of the Hamiltonian at the exceptional point (EP) to ensure it is well-defined and formally consistent, explicitly demonstrating eigenvector coalescence. New section *Exceptional Point and Resolution of Parametrization Singularity* and new Figure S2 have been added in the revised Supplementary Material.

2. In response to Referee #2's comments, we have clarified the discretized nature of our experimental protocol, ensuring it is not misinterpreted as a continuous "in-situ" sweep, and revised the description of error bars.

3. In response to Referee #5's suggestion, we have added an operational sense of adiabaticity to distinguish our results from the conventional Berry-Kato limit.

All the changes and modifications in the revised version of the manuscript and supplementary are highlighted in red text in the additionally submitted file.

In summary, we sincerely thank the referees for their careful and constructive comments. We have revised the manuscript accordingly to address all concerns raised by the referees. We therefore believe that this revised version is now suitable for further consideration in *Nature Communications*.

Report

The authors have carefully addressed not only my previous comments but also those of the other reviewers. Their detailed responses improve the overall clarity of the manuscript, and the experimental realization of the proposed phenomenon is timely and interesting.

After reviewing the revised manuscript, supplementary material, and rebuttal, I am left with one remaining conceptual issue that should be clarified. While the experimental realization and practical aspects of the work are compelling, the remaining issue concerns the conceptual and formal consistency of the underlying Hamiltonian description.

The central claim of the work is that *"we report the first experimental demonstration that the adiabatic state evolution can be fully restored when dynamically encircling an EP, regardless of the winding direction."* This is an important statement, particularly in relation to previous studies on EP encircling and non-adiabatic state switching.

In this context, there is a fundamental ambiguity in the Hamiltonian formulation. According to Eqs. (3)–(5), the EP is identified at the condition $\alpha = 0$. However, at the same point, ϕ is not well-defined, and the present parametrization becomes ill-defined. As a result, the Hamiltonian in Eq. (4), which is expressed in terms of α and ϕ, involves a product of vanishing and diverging quantities, making it unclear whether the Hamiltonian is well-defined exactly at the EP.

If the Hamiltonian is not well-defined at that point, it becomes difficult to rigorously establish that the point itself is an exceptional point of the model. For example, key EP properties such as eigenvector coalescence and self-orthogonality,

$$\langle \eta_{\mathrm{EP}} \mid \psi_{\mathrm{EP}} \rangle = 0, \tag{1}$$

are not explicitly demonstrated in a well-defined manner within the current parametrization.

At the same time, the dynamical phenomena observed when encircling the region are likely consistent with known EP-related behavior. This mismatch between the ill-defined Hamiltonian at the EP and the observed EP-like behavior creates a conceptual ambiguity: if the Hamiltonian is not well-defined at the EP itself, it becomes unclear whether the system encircles a well-defined EP.

This issue could be addressed in several ways. For example:

1. The authors could provide an equivalent Hamiltonian representation that remains well-defined at the EP, and reformulate their analysis accordingly.

2. Alternatively, they could explicitly clarify that the present parametrization becomes

ill-defined at the EP, and explain why the observed dynamics nevertheless reproduces EP-like behavior when encircling the region. In particular, the origin of the EP-like topology and state evolution in the absence of a well-defined Hamiltonian at the EP should be clearly articulated.

I believe that this issue should be addressed before the manuscript can be considered for publication.